

# THE ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and  
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## CONTENTS

### NUMBER 1

A PHOTOELECTRIC STUDY OF VW CEPHEI. C. M. Huffer . . . . .	1
MEASUREMENTS IN THE SPECTRUM OF R HYDRAE. Paul W. Merrill . . . . .	6
THE SPECTROSCOPIC BINARY $\eta$ ANDROMEDAE. Katherine C. Gordon . . . . .	13
THE SPECTROSCOPIC ORBIT OF $\iota$ ARIETIS. Katherine C. Gordon . . . . .	16
THE INFRARED SPECTRUM OF THE MOON. Arthur Adel . . . . .	19
RED MAGNITUDES OF THE NORTH POLAR SEQUENCE STARS. J. J. Nassau and Virginia Burger . . . . .	25
SPECTROGRAPHIC OBSERVATIONS OF $\mu$ CEPHEI. Dean B. McLaughlin . . . . .	35
THE MOTION OF AN ELECTRON IN THE HARTREE FIELD OF A HYDROGEN ATOM. S. Chandrasekhar and Frances Herman Breen . . . . .	41
SPECTROGRAPHIC OBSERVATIONS OF THE ECLIPSING VARIABLE TT HYDRAE. Jorge Sahade and Carlos U. Cesco . . . . .	71
SPECTROGRAPHIC OBSERVATIONS OF ELEVEN ECLIPSING BINARIES. Otto Struve . . . . .	76
THE PERIOD OF THE SPECTRUM VARIABLE $\iota$ CASSIOPEIAE. Armin J. Deutsch . . . . .	99
REVIEWS . . . . .	102
NOTICE . . . . .	104
ERRATA . . . . .	104

### NUMBER 2

ADRIAAN VAN MAANEN. Ralph E. Wilson . . . . .	105
SIX-COLOR PHOTOMETRY OF STARS. IV. THE VARIATION OF $\alpha$ URSAE MINORIS AT DIFFERENT WAVE LENGTHS. Joel Stebbins . . . . .	108
ORBITAL ELEMENTS OF THE ALGOL VARIABLE SS BOÖTIS. Roscoe F. Sanford . . . . .	114
SPECTRA OF BD STARS WITHIN FIVE DEGREES OF THE NORTH POLE. J. J. Nassau and Carl K. Seyfert . . . . .	117
THE SPACE MOTIONS OF THE CLUSTER VARIABLES. Noah W. McLeod . . . . .	134
ON THE EQUATION OF STATE OF IONIZED HYDROGEN. Ralph E. Williamson . . . . .	139 —
THE VARIATIONS OF ABSORPTION-LINE CONTOURS ACROSS THE SOLAR DISC. Merle Tuberg . . . . .	145 —
ON THE RADIATIVE EQUILIBRIUM OF A STELLAR ATMOSPHERE. IX. S. Chandrasekhar . . . . .	165 —
STELLAR MODELS WITH PARTIALLY DEGENERATE ISOTHERMAL CORES AND POINT-SOURCE ENVELOPES. Marjorie Hall Harrison . . . . .	193
THE SPECTRUM OF PROCYON: A TYPICAL STAR OF CLASS F. J. W. Swensson . . . . .	207

## NOTES

AN INTERESTING EMISSION-LINE STAR NEAR THE ORION NEBULA. W. W. Morgan and Stewart Sharpless . . . . .	249
THE SPECTRUM OF HD 151932. Jorge Sahade . . . . .	250
NOTE ON THE PERIOD OF U CEPHEI. John G. Phillips and Arne Slettebak . . . . .	251
REVIEWS . . . . .	252

## NUMBER 3

— THE CORRELATION OF MAGNETIC DISTURBANCES WITH INTENSE EMISSION REGIONS OF THE SOLAR CORONA. A. H. Shapley and W. O. Roberts . . . . .	257
ATOMIC LINES IN THE SPECTRUM OF R LEONIS. Paul W. Merrill . . . . .	275
THE PHOTOGRAPHIC DETERMINATION OF STELLAR PARALLAXES WITH THE 60- AND 100-INCH REFLECTORS. TWENTIETH SERIES. Adriaan van Maanen . . . . .	289
THE ECLIPSING STAR RT ANDROMEDAE. Cecilia Payne-Gaposchkin . . . . .	291
THE SYSTEM OF RX CASSIOPEIAE. Cecilia Payne-Gaposchkin . . . . .	299
ON THE NATURE OF THE ECLIPSES OF $\zeta$ AURIGAE. Zdeněk Kopal . . . . .	310
— APPLICATION OF THE MULTIPLIER PHOTOTUBE TO ASTRONOMICAL PHOTOELECTRIC PHOTOMETRY. Gerald E. Kron . . . . .	326
— NEAR THERMODYNAMIC RADIATIVE EQUILIBRIUM. L. G. Henyey . . . . .	332
— ON THE RADIATIVE EQUILIBRIUM OF A STELLAR ATMOSPHERE. X. S. Chandrasekhar . . . . .	351
THE SPECTRUM OF CLUSTER-TYPE CEPHEIDS. Guido Münch and Luis Rivera Terrazas . . . . .	371

## NOTES

— MICROWAVE RADIATION FROM THE SUN AND MOON. Robert H. Dicke and Robert Beringer . . . . .	375
THE NEAR-ECLIPSING SPECTROSCOPIC BINARIES AS A MEANS FOR STUDYING PECULIAR PHENOMENA IN CLOSE DOUBLE STARS. Gerald E. Kron . . . . .	376
NEW $H\alpha$ EMISSION STARS. W. W. Morgan and W. P. Bidelman . . . . .	378
Be STARS SHOWING BRIGHT LINES OF $Fe$ II. W. W. Morgan and Irene Hansen . . . . .	379
ON THE POLARIZATION OF THE CONTINUOUS RADIATION OF EARLY-TYPE STARS. Edith M. Janssen . . . . .	380
REVIEWS . . . . .	381
ERRATA . . . . .	383
NOTICE . . . . .	384
INDEX . . . . .	385

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THE SPECTROSCOPIC BINARY, ANDROMEDAE . . . . .	Richard G. Aitken	13
THE SPECTROSCOPIC ORBIT OF $\epsilon$ ARIETIS . . . . .	Katherine G. Aitken	19
THE INFRARED SPECTRUM OF THE MOON . . . . .	Robert A. M. S. S. S.	25
RED MAGNITUDES OF THE NORTH POLAR SEQUENCE STARS . . . . .	J. J. Norton and Virginia Searcy	31
SPECTROGRAPHIC OBSERVATIONS OF $\mu$ CEPHEI . . . . .	Don S. McLaughlin	37
THE MOTION OF AN ELECTRON IN THE HARTREE FIELD OF A HYDROGEN ATOM . . . . .	S. Chandrasekhar and P. G. L. S. S.	43
SPECTROGRAPHIC OBSERVATIONS OF THE ECLIPSING VARIABLE, $\gamma$ SCORPIONIS . . . . .	Jojo Saldaña and Carlos U. S. S.	49
SPECTROGRAPHIC OBSERVATIONS OF ELEVEN ECLIPSING BINARIES . . . . .	Jojo Saldaña	55
THE PERIOD OF THE SPECTRUM VARIABLE, CASSIOPEAE . . . . .	Jojo Saldaña	61
REVIEWS . . . . .		
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## A PHOTOELECTRIC STUDY OF VW CEPHEI

C. M. HUFFER

Washburn Observatory

Received November 12, 1945

### ABSTRACT

Two sets of photoelectric measures of VW Cephei, one made in 1932 and the other in 1941-1942, were used for a study of this W Ursae Majoris type eclipsing star. The period is variable, and the light-curve shows continual variation. Since the eclipses are too shallow for a unique solution, an assumption of the relative brightness of the two members of the system was made and the resulting elements obtained.

There have been two important papers about the system of the eclipsing star VW Cephei of the W Ursae Majoris type since its discovery by Schilt in 1927. The first was by Dugan<sup>1</sup> in 1933; the second, by Edith Jones Woodward<sup>2</sup> in 1942. Dugan recognized the difficulty of determining the period but found that Van Gent's<sup>3</sup> elements with double the previous period satisfied his own observations well enough. The elements used by Dugan were

$$\text{Min.} = \text{JD } 2424658.7581 + 0^d27831948 \text{ E.}$$

Mrs. Woodward realized that the period is variable and used 0.27831881 day to represent a series of observations by Kurt Walter in 1929. For her own 1938-1939 observations she used 0.27832156 day. After studying the physical nature of the system, she concluded with the remark: "VW Cephei should certainly be studied photoelectrically."

Two series of photoelectric measures are on file in the records of the Washburn Observatory, publication having been delayed because of pressure of other work. The first series was made in 1932, shortly after the first application of a vacuum-tube amplifier to photoelectric photometry. This star and W Ursae Majoris were used as a test of the amplifier for photometry of stars fainter than those in reach of our former photometer. The study of W Ursae Majoris was published in 1934.<sup>4</sup>

A second series of measures of VW Cephei was made in 1941 and on one night in 1942. This series showed plainly that the period used by Dugan would not fit the observations. Also, an average period satisfying the two series would not fit the minima published by Dugan and Mrs. Woodward. The following formula was used as best representing the published minima and the photoelectric minima, as shown in Table 1:

$$t_1 = \text{JD } 2424658.7574 + \left[ 0^d27831993 + 0^d00000108 \cos \frac{E \cdot 360^\circ}{20,411} \right] \text{ E.} \quad (1)$$

<sup>1</sup> *Contr. Princeton U. Obs.*, No. 13, 1929.

<sup>3</sup> *B.A.N.* 5, 90, 1929.

<sup>2</sup> *Harvard Circ.*, No. 446, p. 19, 1942.

<sup>4</sup> Huffer, *Ap. J.*, 79, 369, 1934.

TABLE 1\*  
OBSERVED MAXIMA AND MINIMA OF VW CEPHEI

No.	E	Observed	Computed	O - C	Authority†
		242			
1.....	0	4658.759	.757	+0.002	Schilt
2.....	79	4680.746	.745	+ .001	Schilt
3.....	132	4695.493	.496	- .003	Van Gent
4.....	135½	4696.472	.470	+ .002	Van Gent
5.....	189½	4711.498	.499	- .001	Van Gent
6.....	354½	4757.425	.422	+ .003	Van Gent
7.....	358	4758.396	.396	.000	Van Gent
8.....	502	4798.475	.475	.000	Van Gent
9.....	894½	4907.715	.716	- .001	Van Gent
10.....	1066½	4955.585	.587	- .002	Van Gent
11.....	2406	5328.404	.397	+ .007	Kukarkin
12.....	2722	5416.348	.346	+ .002	Kukarkin
13.....	3074	5514.322	.315	+ .007	Kukarkin
14.....	3121½	5527.607	.605	+ .002	Dugan
15.....	3122	5527.668	.674	- .006	Dugan
16.....	3186½	5545.629	.626	+ .003	Dugan
17.....	3197	5548.551	.548	+ .003	Dugan
18.....	3208	5551.615	.609	+ .006	Dugan
19.....	3222½	5555.649	.645	+ .004	Dugan
20.....	3240½	5560.651	.655	- .004	Dugan
21.....	3247½	5562.607	.603	+ .004	Dugan
22.....	3283	5572.496	.484	+ .012	Dugan
23.....	3290½	5574.501	.501	.000	Dugan
24.....	3305	5578.599	.607	- .008	Dugan
25.....	3360	5593.911	.914	- .003	Walter
26.....	3498	5632.320	.322	- .002	Oosterhoff
27.....	3790	5713.589	.592	- .003	Kukarkin
28.....	4228	5835.489	.495	- .006	Kukarkin
29.....	4455	5898.674	.674	.000	Dugan
30.....	4993½	6048.614	.618	- .004	Dugan
31.....	5000½	6050.565	.566	- .001	Dugan
32.....	5001	6050.633	.636	- .003	Dugan
33.....	5001½	6050.702	.705	- .003	Dugan
34.....	5004½	6051.540	.540	.000	Dugan
35.....	5004½	6051.606	.610	- .004	Dugan
36.....	5004½	6051.677	.679	- .002	Dugan
37.....	5044½	6062.661	.673	- .012	Dugan
38.....	5069	6069.560	.561	- .001	Dugan
39.....	5069½	6069.628	.631	- .003	Dugan
40.....	5112½	6081.600	.598	+ .002	Dugan
41.....	5205½	6107.620	.621	- .001	Dugan
42.....	7843½	6841.7515	.7534	- .0019	Huffer
43.....	8034	6894.7728	.7729	- .0001	Huffer
44.....	8052	6899.7833	.7826	+ .0007	Huffer
45.....	8059	6901.7342	.7309	+ .0033	Huffer
46.....	8756½	7095.863	.857	+ .006	Kanamori
47.....	10,688	7633.431	.429	+ .002	Walter
48.....	15,852	9070.690	.688	+ .002	Woodward
49.....	16,294	9193.712	.708	+ .004	Woodward
50.....	16,401	9223.493	.488	+ .005	Woodward
51.....	17,239	9456.724	.725	- .001	Woodward
52.....	17,243	9457.828	.838	- .010	Woodward
53.....	17,253	9460.615	.622	- .007	Woodward
54.....	17,268	9464.805	.796	+0.009	Woodward

\* The fraction ½ added to the epoch number (following Dugan's notation) indicates secondary minimum. The fractions ¼ and ¾ indicate maxima.

† For references see Dugan, *op. cit.*

TABLE 1—Continued

No.	E	Observed	Computed	O—C	Authority
		242			
55.....	17,271	9465.631	.631	0.000	Woodward
56.....	17,347	9486.786	.784	+ .002	Woodward
57.....	17,361	9490.682	.681	+ .001	Woodward
		243			
58.....	20,242 $\frac{1}{2}$	0292.6713	.6705	+ .0008	Huffer
59.....	20,310 $\frac{1}{2}$	0311.5963	.5963	.0000	Huffer
60.....	20,411	0339.5679	.5675	+0.0004	Huffer

The fact that the period of the variation (20,411 epochs = 5680 days) has been assumed equal to the number of epochs covered by all the recorded minima of Table 1 indicates that the given formula is intended to represent the observations rather than to determine exactly the period of the variation. The average period is almost exactly the same as that given by Cecilia Payne-Gaposhkin.<sup>5</sup>

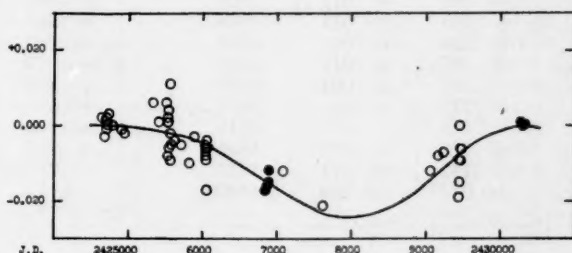


FIG. 1.—Deviations of times of minima of VW Cephei

TABLE 2

HD	Star	R.A. 1900 Decl.	Mag.	Spectrum
197433.....	VW Cephei	20 <sup>h</sup> 38 <sup>m</sup> 6 +75° 13'	8.1–8.4	G5 Shade 0.86
197306.....	BD 75°750	20 37.8 +75 40	8.87	K0
197750.....	BD 75°753	20 40.6 +75 32	8.07	K0

The average period between the first observed minimum of Schilt and the final photoelectric minimum is 0.27832101 day. Figure 1 shows the deviation of the computed times of minima from the average period. The curve was computed by using formula (1). The solid circles stand for the photoelectric minima; the open circles for all the other minima listed in Table 1. It is seen that the scatter of the open circles is considerable, the average deviation from the mean being  $\pm 0.0034$  day. The average deviation of the seven photoelectric minima is  $\pm 0.0010$  day, most of which results from the first and fourth minima of 1932.

Only one comparison star, HD 197750, was used, except on one night, when HD 197306 was used by mistake. In spite of the suspected variation of HD 197750,<sup>6</sup> there

<sup>5</sup> *Pub. A.A.S.*, **10**, 127, 1941.

<sup>6</sup> Dugan, *A.N.*, **254**, 399, 1934.

was no certainty of variation of this star from night to night, and no corrections were applied. However, an empirical nightly correction would improve somewhat the individual residuals but would not affect the normals appreciably.

A neutral-shade glass was used with VW Cephei to equalize more nearly the intensity with the comparison star. Because of the low intensities (nearly ninth photoelectric mag-

TABLE 3  
NORMAL MAGNITUDES OF VW CEPHEI

Phase	Diff. of Mag.	Residual	Phase	Diff. of Mag.	Residual
0 <sup>h</sup> 0138 . . . . .	-0 <sup>m</sup> 004	-0 <sup>m</sup> 001	0 <sup>h</sup> 5156 . . . . .	+0 <sup>m</sup> 073	-0 <sup>m</sup> 024
.0332 . . . . .	+ .001	- .030	.5344 . . . . .	+ .135	+ .018
.0600 . . . . .	+ .084	- .022	.5495 . . . . .	+ .157	+ .017
.1014 . . . . .	+ .184	- .038	.5720 . . . . .	+ .194	+ .011
.1331 . . . . .	+ .266	- .014	.6040 . . . . .	+ .264	+ .018
.1664 . . . . .	+ .310	- .020	.6559 . . . . .	+ .337	+ .016
.1982 . . . . .	+ .399	+ .030	.6920 . . . . .	+ .361	- .002
.2217 . . . . .	+ .372	- .015	.7118 . . . . .	+ .353	- .024
.2407 . . . . .	+ .383	- .012	.7326 . . . . .	+ .383	- .005
.2741 . . . . .	+ .408	+ .015	.7506 . . . . .	+ .366	- .024
.2965 . . . . .	+ .403	+ .024	.7905 . . . . .	+ .388	+ .016
.3181 . . . . .	+ .358	- .001	.8265 . . . . .	+ .352	+ .018
.3463 . . . . .	+ .340	+ .016	.8600 . . . . .	+ .279	- .007
.3661 . . . . .	+ .297	+ .002	.8850 . . . . .	+ .258	+ .011
.3913 . . . . .	+ .222	- .034	.9058 . . . . .	+ .192	- .014
.4143 . . . . .	+ .213	- .001	.9218 . . . . .	+ .173	+ .014
.4364 . . . . .	+ .170	- .000	.9446 . . . . .	+ .097	+ .013
.4635 . . . . .	+ .142	+ .021	.9732 . . . . .	+ .041	+ .029
0.4970 . . . . .	+0.072	-0.019	0.9930 . . . . .	-0.004	+0.004

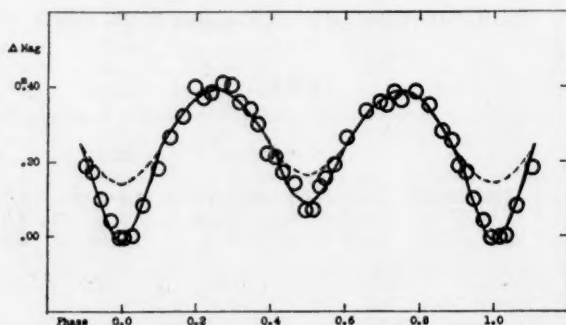


FIG. 2.—Normal magnitudes of VW Cephei

nitude), the photometer was run at full sensitivity for the 1932 measures. The accuracy is therefore less than for the 1941 measures, the amplifier having been improved during the interval. The measures were, nevertheless, given equal weights in forming the normals.

Table 3 gives the normal points obtained by averaging the observed magnitudes in the usual way, 164 observations being combined into 38 normal places. Figure 2 is a graphical representation of the normals from Table 3. The curve is drawn from a formula ob-

tained by a least-squares solution, using 21 normal points between eclipses, with the addition of a curve at minima computed by the Russell method. This formula is

$$\Delta \text{ mag.} = 0^{\text{m}}2724 + 0^{\text{m}}0033 \sin \theta - 0^{\text{m}}0105 \cos \theta - 0^{\text{m}}1212 \cos 2\theta .$$

$$\pm 52 \qquad \pm 66 \qquad \pm 36 \qquad \pm 77$$

The term in  $\cos 2\theta$  rectifies the curve for ellipticity of the figure of the pair of stars. The term  $0^{\text{m}}0033 \sin \theta$  corrects for the difference in maxima. The term,  $-0^{\text{m}}0105 \cos \theta$ , is "reflection effect." The probable error of a single normal magnitude is  $\pm 0^{\text{m}}014$ , which is three or four times that of the brighter eclipsing binaries of our former publications.

After rectification, using the above formula, the curve shows two shallow eclipses, of 0.153 mag. and 0.071 mag., respectively. Since such shallow eclipses do not permit a

TABLE 4  
ELEMENTS OF VW CEPHEI

Amount of eclipse, larger body.....	$a_0''$	0.106
Amount of eclipse, smaller body.....	$a_0$	0.328
Ratio of radii.....	$k$	0.567
Loss of light at $t_1$ .....	$1-\lambda_1$	0.1314
Loss of light at $t_2$ .....	$1-\lambda_2$	0.0633
Light of larger body.....	$L_1$	0.600 (assumed)
Light of smaller body.....	$L_2$	0.400
Ratio of surface brightness.....	$J_2/J_1$	0.482
Inclination of orbit.....	$i$	$63^{\circ}0$
Radii of larger body.....	$\begin{cases} a_1 \\ b_1 \end{cases}$	$\begin{cases} 0.525 \\ 0.398 \end{cases}$
Radii of smaller body.....	$\begin{cases} a_2 \\ b_2 \end{cases}$	$\begin{cases} 0.297 \\ 0.226 \end{cases}$
Ellipticity constant.....	$\varepsilon$	0.337
Eccentricity of figure.....	$e$	0.652
Difference of magnitude.....		$0^{\text{m}}44$

unique solution, a solution which would satisfy the requirements for two spectra having intensity ratio of two to three was made. This forced solution gave smaller residuals than a solution obtained without previous assumptions and hence was the most satisfactory tried.

Table 4 shows the elements of VW Cephei obtained from this solution. Since there is no available spectrographic orbit, the absolute elements are not obtainable.

It is suggested that this interesting eclipsing star be studied spectroscopically. With our present photoelectric equipment it would not be possible to improve on the light-curve. A few more determinations of the time of minimum should improve the formula which has been deduced in this investigation.

It is a pleasure to acknowledge assistance from Messrs. Joel Stebbins and Albert E. Whitford. The photometer was built by the latter, work on which was supported in part by a grant from the National Research Council.

## MEASUREMENTS IN THE SPECTRUM OF R HYDRAE\*

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### ABSTRACT

Measurements of numerous atomic lines in the region  $\lambda$  3700 to  $H\beta$ ,  $\lambda$  4860, were made on spectrograms of dispersion 10 Å/mm taken chiefly during the declining phase of the light-cycle. The average displacements of the *dark lines* of various metals are found to agree well, a slight exception being offered by lines of K. The velocity-curve with phase has considerable resemblance to the light-curve. The mean displacement of Fe I lines of E.P. 1.6 volts is algebraically greater by about 4 km/sec than that of lines of E.P. 0.0 volts.

*Bright lines* of H, Fe I, Fe II, [Fe II], Mg I, Si I, and Ca II have been measured. Changes in their displacements with phase in the light-cycle are intercompared in a series of curves. The Fe I line  $\lambda$  3852 and the unidentified line  $\lambda$  4373 are discussed in some detail. After maximum light, diffuse emission appears on the shortward sides of H and K. It is stronger at H than at K.

The star R Hydrae, 132422, spectrum M6e, is a well-known long-period variable, discovered by G. P. Maraldi in 1704. Its period has undergone a decrease, apparently secular, from about 500 days in the eighteenth century to about 390 days at present. No other long-period variable is known to have changed its period so markedly, although one other, R Aquilae, has shown a definite (but considerably smaller) decrease.

The present *Contribution*, which is the second in a new series on the spectra of long-period variables,<sup>1</sup> presents a brief study of the atomic lines in the region  $\lambda$  3700 to  $H\beta$  ( $\lambda$  4860) of the spectrum of R Hydrae, based on coudé spectrograms, dispersion 10.3 Å/mm, taken chiefly during the declining phase of two successive light-cycles. Dates of observation are in Table 1, and their relationship to the light-curve is shown in Figure 1. The photometric data were supplied through the courtesy of Mr. Leon Campbell, of the Harvard Observatory.

### ABSORPTION LINES

About six spectrograms showing the dark lines were obtained in each of the years 1943 and 1944, and a single one in 1945. As far as may be judged from this rather meager material, the behavior of the absorption spectrum at maximum light and during the decline was similar in the three cycles.

Near maximum light the absorption spectrum is of class M6, closely resembling that of  $\alpha$  Ceti at the same phase. The observed changes with phase, for the most part, do not affect very radically the general appearance of the spectrum. Certain details, however, exhibit rather striking variations (see Pls. I-IV). The H and K lines of Ca II, wide and strong at maximum, become less intense during the decline of light; moreover, they develop very curious emission components in the shortward<sup>2</sup> wings (see p. 12). The lines  $\lambda$  4077,  $\lambda$  4215 of Sr II also are stronger at maximum, while ultimate lines of neutral metals, notably  $\lambda$  4227 Ca I;  $\lambda$  4044,  $\lambda$  4047 K I; and  $\lambda\lambda$  4254, 4274, 4289 Cr I are weaker. The line  $\lambda$  4172 exhibits a marked decrease in intensity after maximum; detailed discussion of this line is reserved for a future *Contribution*.

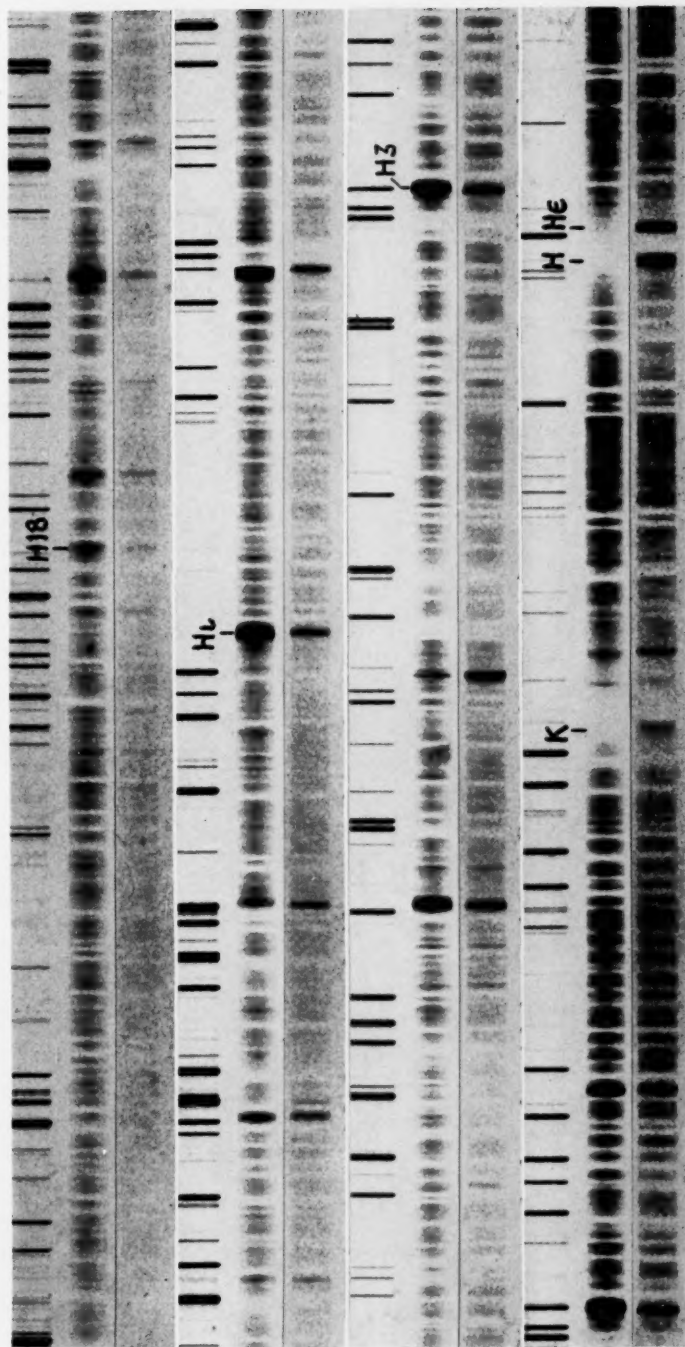
The residuals in displacements yielded by lines of various chemical elements (Table 2) are much like those found in *Mount Wilson Contributions*, No. 713<sup>1</sup> for the four stars U Ori, R Ser, R Aql, and R Cas; and the remarks there apply also to R Hya. The dependence of line displacement on excitation potential, previously observed in the spectra of

\* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 717.

<sup>1</sup> The first was *Mt. W. Contr.*, No. 713; *A. J.*, **102**, 347, 1945.

<sup>2</sup> A coined word for "toward shorter wave lengths."

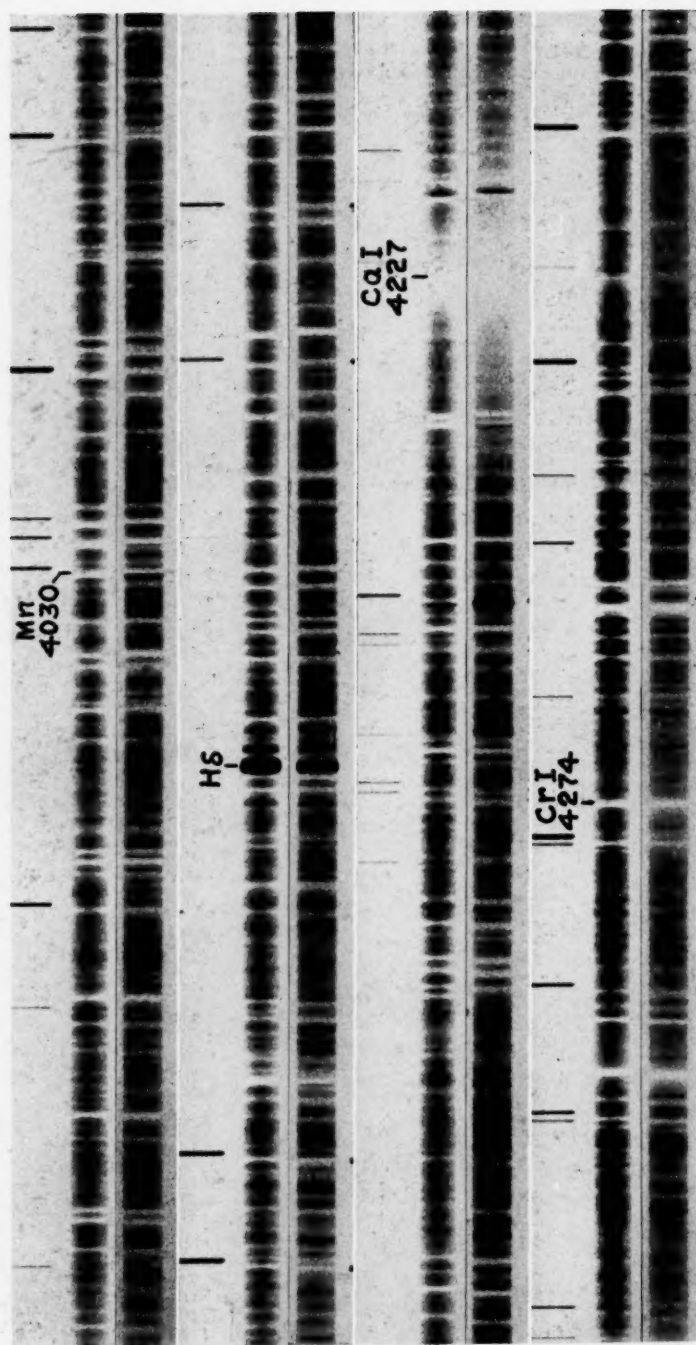
# PLATE I



SPECTROGRAMS OF R HYDRAE,  $\lambda\lambda$  3630-3990

In each pair: *above*, Ce 3390, February 10, 1944, phase - 19 days; *below*, Ce 3419, May 10, 1944, phase + 71 days

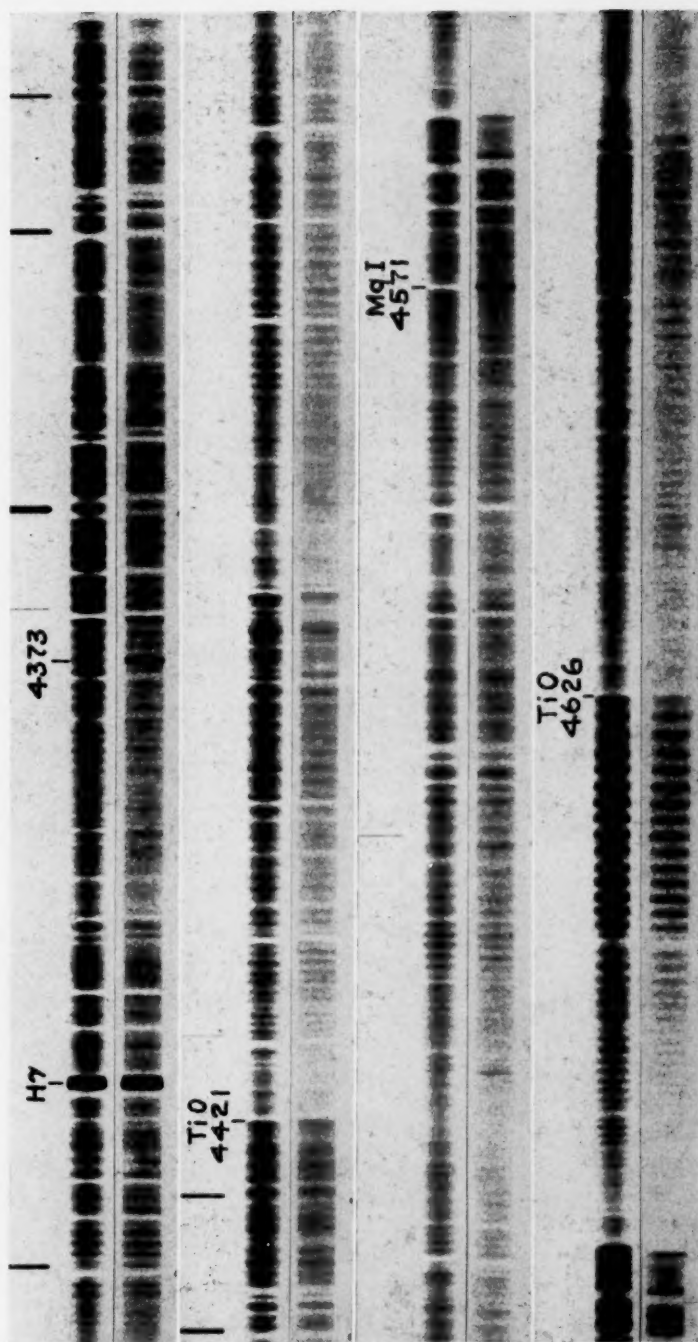
# PLATE II



SPECTROGRAMS OF R HYDRAE,  $\lambda\lambda$  3970-4330

In each pair: *above*, Ce 3390, February 10, 1944, phase -19 days; *below*, Ce 3419, May 10, 1944, phase +71 days

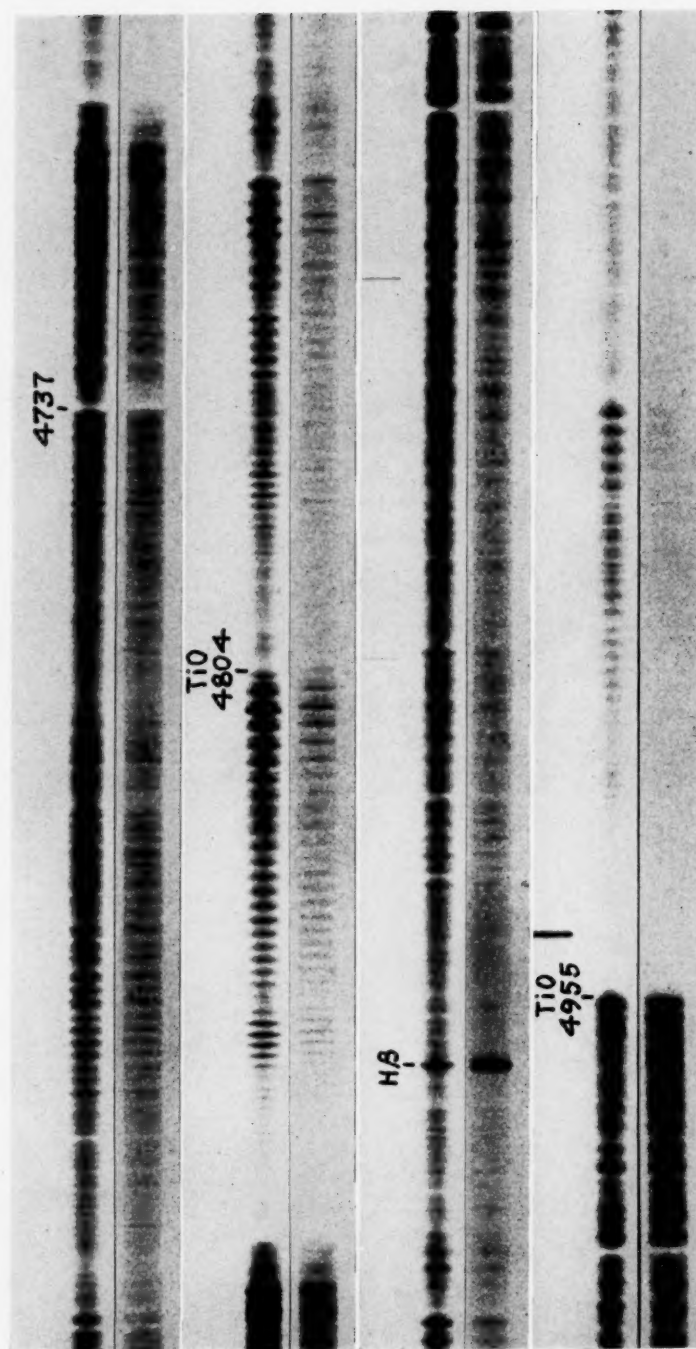
# PLATE III



SPECTROGRAMS OF R HYDRAE,  $\lambda\lambda$  4320-4680

In each pair: *above*, Ce 3390, February 10, 1944, phase - 19 days; *below*, Ce 3419, May 10, 1944, phase + 71 days

# PLATE IV



SPECTROGRAMS OF R HYDRAE,  $\lambda\lambda$  4670-5030

In each pair: *above*, Ce 3390, February 10, 1944, phase -19 days; *below*, Ce 3419, May 10, 1944, phase +71 days

TABLE 1  
JOURNAL OF OBSERVATIONS

PLATE Ce	DATE	JD 243	MAG.	PHASE (DAYS)	RAD. VEL. FROM ABS. LINES	
					Km/Sec	No. Lines
2767.....	1942 Apr. 4	0454	6.6	+ 64	.....	.....
2959.....	1943 Feb. 25	0781	4.7	+ 21	- 8.2	64
2971.....	Mar. 23	0807	5.9	+ 47	-10.2	28
2974.....	Mar. 24	0808	5.9	+ 48	- 9.0	41
3018.....	Apr. 24	0839	6.8	+ 79	-12.4	26
3030.....	May 18	0863	7.6	+103	-14.8	10
3052.....	May 25	0870	7.7	+110	.....	.....
3057.....	June 9	0885	8.0	+125	-15.6	22
3076.....	June 22	0898	8.3	+138	.....	.....
3362.....	1944 Jan. 15	1105	5.9	- 45	- 6.1	48
3390.....	Feb. 10	1131	4.7	- 19	- 9.1	72
3394.....	Feb. 11	1132	4.7	- 18	.....	.....
3402.....	Apr. 7	1188	5.8	+ 38	-10.8	30
3419.....	May 10	1221	6.9	+ 71	-11.8	35
3437.....	June 2	1244	7.4	+ 94	.....	.....
3438.....	June 3	1245	7.4	+ 95	-16.1	24
3442.....	June 4	1246	7.5	+ 96	(-14.3)	12
3496.....	July 8	1280	8.4	+130	.....	.....
3756.....	1945 Apr. 27	1573	6.0	+ 33	- 7.7	50

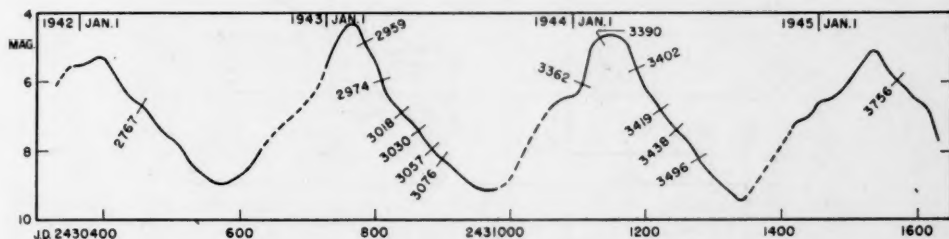


FIG. 1.—Light-curve of R Hydrae showing dates of spectrograms

TABLE 2  
DISPLACEMENTS OF ABSORPTION LINES  
(Residuals in Km/Sec)

Element	R Hya	Four Me Stars	Element	R Hya	Four Me Stars
Al I.....	-0.2	.....	Mn I.....	(+0.5)	(-0.6)
K I.....	-3.3	-3.3	Fe I.....	+0.8	+ .4
Sc I.....	+2.0	+0.5	Co I.....	-0.1	.0
Ti I.....	-1.0	-0.1	Ni I.....	+0.5	+ .2
V I.....	-0.2	-0.5	Sr II.....	(+1.7)	(-0.5)
Cr I.....	-1.1	-1.1			

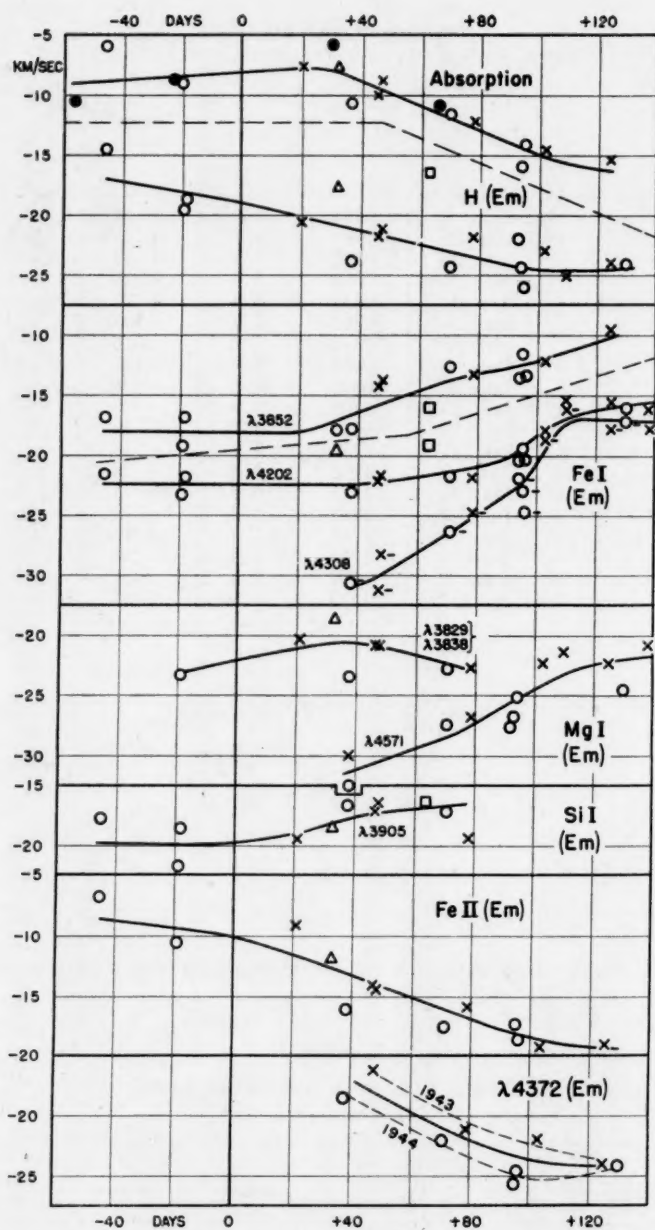


FIG. 2.—Velocity-curves derived from absorption and emission lines. Observations in various years are indicated as follows: squares, 1942; crosses, 1943; open circles, 1944; triangles, 1945; solid circles, normal places from Table 2, *Mt. W. Contr.*, No. 644; *A p. J.*, 93, 380, 1941.

other Me variables,<sup>1,3</sup> is exhibited by the iron lines of R Hydrae. On two of the best plates, Ce 3390, 3756, the mean displacement at E.P. 1.6 volts is algebraically greater by about 4 km/sec than at 0.0 volts.

The mean absorption-line displacements adopted for individual plates are in Table 1. Plotted according to phase (Fig. 2), they define a curve, which, after a flat maximum near the time of maximum light, falls slowly during declining light. At phase +120 days (0.31 period) it is 8 or 9 km/sec below the highest velocity. This curve, nearly in phase with the light-curve, resembles that for  $\alpha$  Ceti determined by A. H. Joy,<sup>4</sup> but generalization should be withheld because other data suggest that typical Me variables do not necessarily behave in this manner in all cycles.

## EMISSION LINES

Although the bright hydrogen lines in the spectrum of R Hydrae are very conspicuous during nearly half the whole light-cycle, bright lines of other elements are probably less numerous and less intense than in some other Me variables of about the same period. As a rule, emission lines seem to develop more strongly in spectral types later than M6. In

TABLE 3  
DISPLACEMENTS OF BRIGHT HYDROGEN LINES  
ON PLATE CE 3419, PHASE +71 DAYS

Line	Rad. Vel.	Line	Rad. Vel.
H 18.....	-23.4	H $\eta$ .....	-25.3
H 17.....	-22.6	H $\zeta$ .....	-25.8
H 16*.....	.....	H $\epsilon$ .....	-27.0
H 15.....	-24.9	H $\delta$ .....	-26.5
H 14.....	(-20.9)	H $\gamma$ .....	-24.9
H 13.....	-24.1		
H 12.....	-24.8	Mean (13)....	-24.5 $\pm$ 0.30
H $\epsilon$ .....	-23.9	Mean (12)†...	-24.7 $\pm$ 0.25
H $\theta$ .....	-23.8		

\* Double.

† Omitting H 14.

discussing observations of long-period variables, one should bear in mind possible "meteorological" variations in behavior from one cycle to another. In the present short series of spectrograms of R Hydrae, however, differences between cycles are not very marked.

*Lines of hydrogen.*—Near time of maximum light the bright hydrogen lines are intense and slightly widened but are probably not so wide as in some other Me variables, e.g., U Ori. After maximum they gradually become weaker and narrower, disappearing before minimum light at phase about +140 days. The radial velocity yielded by the bright H lines appears to decrease steadily from -17 km/sec at phase -40 days to -25 km/sec at +120 days. The velocities in 1944 lie slightly below those in 1943 (see Fig. 2). As an example of the accuracy of the measures of the hydrogen velocity during the postmaximum phase, the values for individual lines on plate Ce 3419, phase +71 days, are recorded in Table 3. The internal probable error of the mean is  $\pm 0.30$  km/sec, or slightly smaller if H 14 is omitted. The mean displacements on each plate are in Table 4.

*Lines of Fe I.*—The strongest and most persistent bright lines of Fe I are  $\lambda\lambda$  3852, 4202, and 4308, all well known in Me spectra. Of these,  $\lambda$  4202 is present on every plate of the series;  $\lambda$  3852 on all except three which have insufficient exposure;  $\lambda$  4308, on the other hand, is missing on the earlier plates of each light-cycle. Early in each cycle,  $\lambda$  3852 is the

<sup>3</sup> W. S. Adams, *Mt. W. Contr.*, No. 638; *A. J.*, 93, 11, 1941.

<sup>4</sup> *Mt. W. Contr.*, No. 311; *A. J.*, 63, 281, 1926.

strongest of the three lines; at phase +70 to +80 days it is passed by  $\lambda$  4202; still later, at +90 to +100 days,  $\lambda$  4308 becomes the strongest.

The velocity-curve derived from  $\lambda$  4202 (Fig. 2) is nearly flat for a considerable interval, with a marked rise beginning about phase +80 days. In the early part of the cycle the measured velocity may be decreased slightly by the presence of an absorption component on the longward side of the bright line, but the shift is probably not very great. The effect is much more pronounced in the measured displacement of  $\lambda$  4308 when this line first appears at phase about +40 days. When the absorption components become weak, both bright lines yield nearly the same displacement.

The velocity-curve derived from  $\lambda$  3852 approximately parallels that for  $\lambda$  4202 but is about 4-6 km/sec (0.07 A) higher. This relative displacement may be caused by blending

TABLE 4  
DISPLACEMENTS OF EMISSION LINES (KM/SEC)  
(All Values Are Negative)

PLATE Ce	PHASE (DAYS)	H	Si I	Fe I			Mg I				In I	Fe II	Fe [II]
			$\lambda$ 3905	$\lambda$ 3852	$\lambda$ 4202	$\lambda$ 4308	$\lambda$ 3829	$\lambda$ 3832	$\lambda$ 3838	$\lambda$ 4571	$\lambda$ 4511		
2767...	+ 64	16.6	16.5	16.0	19.1								
2959...	+ 21	20.7	19.6	(20.9)			21.2	25.1	19.6				(9.3)
2971...	+ 47	22.0	17.2	14.4	22.2	(31.4)	19.3	21.7	22.3				14.4
2974...	+ 48	21.4	16.6	13.9	21.7	28.3	22.6	25.0	19.4	(30.0)	16.0		14.6
3018...	+ 79	22.0	19.5	13.5	21.9	24.9	24.5	26.1	22.1	26.9	13.9		16.0
3030...	+103	23.2		12.2	18.0	18.7				22.3			19.4
3052...	+110	(25.2)			15.5	16.2				21.5			
3057...	+125	(24.1)		9.6	15.6	17.8				22.4		(19.2)	16.4
3076...	+138				16.2	17.8				20.9			
3362...	- 45	14.8	17.8	16.9	21.6			25.8			15.4	( 6.9)	
3390...	- 19	19.7	21.8	19.3	23.3		23.5	28.9	24.1			10.7	
3394...	- 18	18.9	18.7	16.9	(21.9)								
3402...	+ 38	24.0	16.8	17.8	23.1	30.7	23.5	28.2	23.4	32.5	17.1	16.4	
3419...	+ 71	24.5	17.3	12.7	21.8	26.4	23.7	27.6	22.1	27.5	12.4	17.7	
3437...	+ 94	(21.6)		(13.6)	(20.5)	(17.0)				(27.7)			
3438...	+ 95	24.6		(11.6)	19.5	18.1				26.8		(17.0)	(17.0)
3442...	+ 96	26.2		13.5	20.5	19.8				25.1	(17.8)	(18.6)	(25.3)
3496...	+130	(24.2)			16.2	17.2				24.7			(22.0)
3756...	+ 33	17.8	18.6	18.0	19.7		(18.1)	(25.9)	(18.8)		12.0	9.3	

with a close companion. On several plates the line appears unsymmetrical, as if partially resolved, the weaker side being longward. If settings are made on the maximum, the displacement comes out about the same as that of  $\lambda$  4202. On plate Ce 3390, phase -19 days, the components, both weak and sharp, are well separated, with a measured wavelength interval of 0.38 A. The apparent wave length of the faint component would then be  $Fe\ I\ \lambda\ 3852.57 + 0.38 = 3852.95$ . The faint component seems to gain in relative intensity as the phase advances; it may be this behavior that gives the velocity-curve a slightly different shape from that of  $\lambda$  4202. On the whole, the new data tend to substantiate the identification of the main component with  $Fe\ I\ \lambda\ 3852.57$ . The identification of the companion line is not obvious.

The low-temperature lines  $\lambda\lambda$  4206, 4216, 4375, 4427, 4461, E.P. 0.0 volts, often observed in emission in the postmaximum spectrum of other Me variables, exhibit little emission in R Hydrae, although well marked in absorption. In fact, except at  $\lambda$  4376, where the dark line has a weak shortward bright edge on a few postmaximum plates, no emission is clearly seen in the present series.

*Lines of Mg I.*—The ultraviolet triplet  $\lambda\lambda$  3829, 3832, 3838 appears weakly on nearly all the well-exposed plates. The displacements of  $\lambda$  3829 and  $\lambda$  3838 agree rather closely with those of the *H* lines on the same plates. As in other Me stars,<sup>1</sup>  $\lambda$  3832 has a shortward relative displacement of a few km/sec (Table 4).

The line  $\lambda$  4571 appears 40 or 50 days after maximum light, increasing in intensity thereafter. Its behavior is much like that of the *Fe I* line  $\lambda$  4308, except that its greatest intensity occurs somewhat later in the light-cycle. The velocity-curve (Fig. 2), also resembles that of  $\lambda$  4308 but lies a few km/sec lower.

*Lines of Si I.*—The line  $\lambda$  3905 appears in moderate intensity on all plates except a few taken at the most advanced phases. The measured displacements have a mean value of  $-18.2$  km/sec; the variation with phase is small (Fig. 2). The line  $\lambda$  4103, although present on a few plates, is not well marked, and measured displacements are of little value.

*Lines of Fe II.*—The lines  $\lambda\lambda$  3938, 4178, and 4233 were measured on most of the well-exposed plates. The velocity-curve (Fig. 2) resembles that of hydrogen in that it falls steadily during each cycle. At phase  $-40$  days, however, it lies 9 km/sec above that for

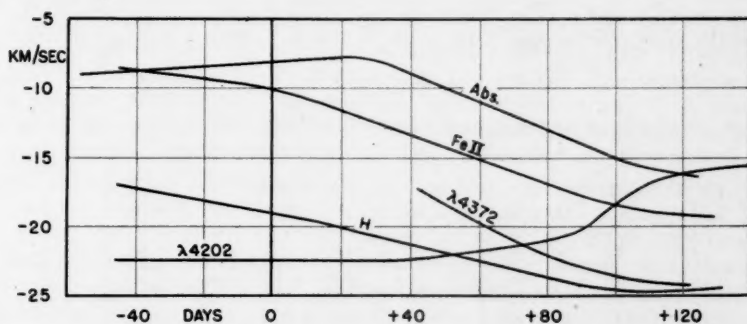


FIG. 3.—Comparison of velocity-curves from various lines

hydrogen; at  $+120$  days, 5 km/sec above. It thus lies mainly between the curve for hydrogen and that for the absorption lines (Fig. 3). Somewhat similar behavior was noticed years ago by A. H. Joy in the spectrum of  $\alpha$  Ceti.<sup>4</sup>

A few of the strongest forbidden lines, [*Fe II*], were measured at advanced phases (Table 4). The displacements were not measurable with high accuracy but probably agree with those of permitted lines of *Fe II* on the same plates. Toward minimum the [*Fe II*] lines become outstanding.<sup>5</sup>

*The line  $\lambda$  4373.*—This unidentified emission line develops 40 or 50 days after maximum, has its greatest intensity a month or two later, say at phase  $+100$  days, but is never nearly so strong as  $\lambda$  4202 or  $\lambda$  4308. On the assumption that its velocity displacement corresponds to that read from the curve for  $\lambda$  4202 (Figs. 2, 3), its mean wave length is 4372.58, which differs very little from the values previously determined with low dispersion in various Me stars by A. H. Joy<sup>4</sup> and by the writer.<sup>6</sup> In the present series of values there is a well-defined progression with phase from  $\lambda$  4372.69 at  $+40$  days to  $\lambda$  4372.49 at  $+130$  days. The shift represents a change in apparent velocity differing considerably from that exhibited by  $\lambda$  4202. On the assumption that the velocity corresponds to that read from the curve for hydrogen, the mean wave length is  $\lambda$  4372.63, the shift being from  $\lambda$  4372.68 at  $+40$  days to  $\lambda$  4372.61 at  $+130$  days. As far as displace-

<sup>5</sup> See P. W. Merrill, *Spectra of Long-Period Variable Stars*, Pl. IV, opp. p. 54, Chicago, 1940.

<sup>6</sup> *Mt. W. Contr.*, No. 399; *Ap. J.*, 71, 285, 1930.

ment is concerned, the behavior is somewhat like that of *H* or *Fe* II, but the drop is even greater, the curve nearly paralleling that for the absorption lines. It is curious that the velocity-curve, based on a constant wave length (Figs. 2, 3), resembles the curves for *H* and *Fe* II more than it does those for the postmaximum lines of neutral metals. Possibly the line is due to an ionized atom. Its mean wave length referred to lines of *Fe* II would be about  $\lambda$  4372.54.

*Comparison of velocity-curves from various lines.*—To facilitate comparison, several curves plotted separately in Figure 2 are brought together on the same set of co-ordinates in Figure 3. The lines of neutral atoms, exemplified by *Fe* I  $\lambda$  4202, seem to have a behavior systematically different from that of bright lines of *H* and *Fe* II and from that of absorption lines. The interpretation of these complex displacements, as yet quite obscure, will doubtless be aided by comparable observations of other Me variables. For this purpose spectrograms of R Leonis have been obtained; their measurement and discussion are under way.

Since the normal wave length of  $\lambda$  4372 is unknown, its position on the velocity-scale is arbitrary; perhaps it belongs about 5 km/sec higher, in which case it would nearly coincide with the curve for *Fe* II.

*Displaced emission of Ca II.*—About 40 days after maximum, diffuse bright lines appear within the shortward wings of the dark H and K lines of *Ca* II and become stronger

TABLE 5  
DISPLACEMENTS OF BROAD *Ca* II EMISSION LINES WITH RESPECT TO DARK LINES

Plate Ce	Phase (Days)	H (Km/Sec)	K (Km/Sec)	Plate Ce	Phase (Days)	H (Km/Sec)	K (Km/Sec)
2971.....	+ 47	-95	.....	3402.....	+38	-97	.....
2974.....	+ 48	-87	.....	3419.....	+71	-75	-96
3018.....	+ 79	-72	-90	3438.....	+95	-50	.....
3057.....	+125	-42	-50	3442.....	+96	-63	.....

as the phase advances. These lines, previously observed in the spectrum of  $\alpha$  Ceti by Joel Stebbins<sup>7</sup> and in several other Me variables by the writer,<sup>6</sup> have not heretofore been identified. It now appears probable that they are portions of the complex H and K lines.

The emission appears only on the shortward edge of the central cores of H and K, recalling the fact that emission lines of *Fe* I and *Mg* I, in their initial stages, are bright edges on the shortward sides of dark lines. The *Ca* II bright lines are more diffuse than bright lines of other metals; and their effective centers, measurable with reasonable accuracy, are much more greatly displaced (Table 5). As the phase advances, the centers shift longward. Their behavior is thus qualitatively similar to that of emission lines of *Fe* I.

These emission portions of H and K remotely resemble the H2 and K2 emission well known within the dark *Ca* II lines of disturbed areas on the sun. Many late-type stars exhibit reversals in the *Ca* II lines more or less like those in solar spectra; in most of these the emission when unsymmetrical is stronger on the *longward* side of the central dark core. In the spectrum of  $\beta$  Draconis, cG2, the *shortward* portion is the stronger,<sup>8</sup> but the emission is weaker than that in R Hydrae and the similarity is not close.

A very curious fact is that the emission at K is much weaker than that at H, and more greatly displaced. This phenomenon seems not to have any parallel in the reversals found in the sun or in nonvariable late-type stars and must therefore be due to some special circumstances in the atmospheres of Me stars.

<sup>7</sup> *Lick Obs. Bull.*, 2, 78, 1903.

<sup>8</sup> Adams and Joy, *Pub. A.S.P.*, 43, 407, 1931.

# THE SPECTROSCOPIC BINARY $\eta$ ANDROMEDAE\*

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## ABSTRACT

The orbital elements of both G-type components of the spectroscopic binary  $\eta$  Andromedae have been determined. The stars move in nearly circular orbits with a period of 115.7 days. The ratio of the masses of the two components was found to be 1.11.

The star  $\eta$  Andromedae ( $\alpha = 0^h 51^m 9$ ,  $\delta = +22^\circ 53'$ , 1900; vis. mag. 4.62) was announced as a spectroscopic binary, with the spectra of both components visible, in 1900 by W. W. Campbell and W. H. Wright.<sup>1</sup> The spectral type is G5, according to the *Henry Draper Catalogue* and also in the Mount Wilson classification. Spectrograms show that the fainter component is of similar spectral type.

Fifty spectrograms were available for the determination of the spectroscopic orbit. They were taken between October, 1899, and August, 1944. The first six were obtained with the original Mills three-prism spectrograph ( $H\gamma$  central); the remainder were obtained with the new Mills three-prism spectrograph ( $\lambda$  4500 central, 11 A/mm).

Table 1 gives the observations and results. The first column gives the Universal Time of the observation, and the second, the Julian Day. The third and fifth columns contain the measured velocities of the brighter and fainter components, respectively. Measures of the first six plates were taken from *Publications of the Lick Observatory*, Vol. XVI. The remaining plates were measured by the writer on the Hartmann spectrocomparator against a standard plate of  $\alpha$  Boötis. The velocity measures needed no reduction to the Lick Observatory system, since the personal equation had been found equal to 0.0 km/sec. Parentheses indicate values not used in the least-squares solution, for reasons stated later.

A period of 115.71 days was determined from a plot of the velocities, and an ephemeris was computed on the basis of the following preliminary elements:

$$\begin{array}{ll} T = \text{JD } 2430122.6 & K_1 = 17.5 \text{ km/sec} \\ \omega = 177.1^\circ & K_2 = 19.3 \text{ km/sec} \\ e = 0.1 & V = -10.3 \text{ km/sec} \\ P = 115.71 \text{ days} \end{array}$$

Fifty-three velocity measures were grouped into 27 normal places, 15 for the brighter and 12 for the fainter component, observations of the fainter component being given half-weight. The solution resulted in the following final elements with their probable errors:

$$\begin{array}{ll} T = \text{JD } 2430119.11 \pm 21.93 \text{ days} & V = -10.33 \text{ km/sec} \\ \omega = 166.38 \pm 70.36 & a_1 \sin i = 28.5 \times 10^6 \text{ km} \\ e = 0.005 \pm 0.032 & a_2 \sin i = 31.5 \times 10^6 \text{ km} \\ P = 115.71 \text{ days (assumed)} & m_1 : m_2 = 1.11 \\ K_1 = 17.89 \pm 0.44 \text{ km/sec} & m_1 \sin^3 i = 0.34 \odot \\ K_2 = 19.83 \pm 0.68 \text{ km/sec} & m_2 \sin^3 i = 0.31 \odot \end{array}$$

The probable error of a single observation of unit weight is  $\pm 1.37$  km/sec.

The solution reduced the sum of the squares of the residuals from 39.61 to 34.84. A check on the solution was furnished by the circumstance that in no case did the differ-

\* Contributions from the Lick Observatory, Ser. II, No. 11.

<sup>1</sup> *A. J.*, 12, 256, 1900.

TABLE 1  
RADIAL VELOCITIES OF  $\eta$  ANDROMEDAE

DATE U.T.	JULIAN DAY	BRIGHTER COMPONENT		FAINTER COMPONENT		PHASE (DAYS)	PHOTO- GRAPHED BY*
		V (Km/Sec)	O-C (Km/Sec)	V (Km/Sec)	O-C (Km/Sec)		
1899 Oct. 25.....	2410000+						
Nov. 1.....	4952.85	-25	-0.8	+ 5	-0.1	74.3	W
1900 July 25.....	4959.74	-26	+1.5	+ 9	+0.3	81.2	W
Aug. 9.....	5225.98	(-11.3)	(-1.2)			0.4	W
Sept. 10.....	5240.94	+ 2	-0.7	-30	-5.2	15.2	W
Sept. 18.....	5272.88	- 1.5	-1.1			47.2	W, C
	5280.85	(- 9.5)	(-1.1)			55.2	W
1931 Aug. 2.....	2420000+						
1935 Sept. 17.....	6555.99	(-12.6)	(+6.2)			106.4	P
Oct. 4.....	8062.93	(-10.8)	(+5.6)			109.1	P
1939 Sept. 28.....	8079.87	(- 9.9)	(-9.1)			10.3	M, P
1940 Sept. 23.....	9534.92	-26.4	-0.7	+ 6.6	-0.1	76.9	M, P
	9895.93	-27.6	+0.3	+ 8.1	-1.0	90.7	P
1941 Sept. 5.....	2430000+						
Sept. 13.....	0242.99	-28.8	-0.9	+ 7.3	-1.9	90.7	P
Sept. 17.....	0250.98	-24.6	+0.1	+ 4.0	-1.6	98.7	P
Sept. 24.....	0254.92	-18.6	+3.5	+ 1.6	-1.1	102.6	P
Sept. 28.....	0261.90	(-12.9)	(+2.3)			109.6	P
Oct. 30.....	0265.92	(-11.4)	(+1.1)			113.6	P
Nov. 9.....	0297.82	+ 7.3	-0.2	-31.6	-1.6	29.8	P
Nov. 26.....	0307.79	+ 3.9	-0.7	-23.2	+3.6	39.8	M
Dec. 7.....	0324.71	(-10.2)	(-0.8)			56.7	P
Dec. 8.....	0335.80	(-14.9)	(+4.6)			67.8	Wn
Dec. 8.....	0336.70	(-13.8)	(+6.4)			68.7	M, G
Dec. 8.....	0336.73	(-13.6)	(+6.6)			68.7	M, G
Dec. 8.....	0336.75	(-13.6)	(+6.6)			68.7	M, G
Dec. 11.....	0339.78	-17.4	+5.2	+ 4.0	+0.8	71.8	M, I
1942 Jan. 4.....	0363.69	-26.9	-0.7	+ 7.9	+0.6	95.7	Wn
Jan. 5.....	0364.68	-23.9	+1.9	+ 6.1	-0.7	96.7	Wn
Jan. 9.....	0368.65	-21.0	+2.5	+ 4.4	+0.2	100.6	I
Feb. 10.....	0400.63	(- 0.1)	(-3.8)			16.9	P
Feb. 14.....	0404.66	+ 5.6	-0.2	-26.4	+1.8	20.9	Ne
Feb. 17.....	0407.63	+ 7.5	+0.7	(-28.6)	(+0.6)	23.9	P
Feb. 26.....	0416.62	(+ 5.7)	(-1.4)			32.9	M, Wn
Aug. 25.....	0596.98	-27.4	-2.1	+ 5.9	-0.4	97.5	P
Aug. 27.....	0599.00	-25.9	-1.8	+ 5.7	+0.7	99.5	P
Sept. 10.....	0612.93	(-11.2)	(+1.5)			113.5	Wn
Sept. 11.....	0614.00	(-11.2)	(-0.7)			115.6	Wn
Sept. 20.....	0622.94	(- 8.8)	(-5.9)			7.8	Nt
Sept. 27.....	0629.86	+ 0.9	-1.5	-18.8	+5.6	14.7	Wn
Oct. 3.....	0635.94	+ 6.2	+0.5	-28.3	-0.2	20.8	P
Oct. 13.....	0645.85	+ 8.2	+0.8	-32.4	-2.4	30.7	P
Oct. 17.....	0649.86	+ 7.9	+1.2	(-29.4)	(-0.2)	34.7	P
Oct. 20.....	0652.83	+ 5.6	0.0			37.7	Nt
Oct. 22.....	0654.81	+ 5.5	+0.9	-29.5	-2.6	39.7	Nt
Oct. 29.....	0661.81	- 1.2	-1.2	(-18.5)	(+3.1)	46.7	Nt
Nov. 5.....	0668.81	(- 8.9)	(-2.3)			53.9	P
Nov. 25.....	0688.74	-24.3	+0.4	+ 4.1	-0.5	73.6	Nt
Dec. 3.....	0696.76	-31.9	-4.3	+ 9.3	+0.5	81.6	Nt
Dec. 10.....	0703.68	(-29.7)	(-1.4)			88.5	Nt
Dec. 17.....	0710.70	-27.7	-1.4	+ 9.9	+2.5	95.5	Nt
1943 Jan. 1.....	0725.66	(-12.2)	(+3.1)			110.5	M
1944 Aug. 22.....	1325.02	(- 4.1)	(-6.9)			15.6	P

\* W = Wright; C = Campbell; P = Paddock; M = Moore; Wn = Wirtanen; G = Gordon; I = Irwin; Ne = Neubauer; Nt = Nagtegaal.

ence between the residuals derived from the ephemeris and those obtained by substitution into the 27 equations of condition exceed 0.13 km/sec. The residuals from the ephemeris are listed in the fourth and sixth columns of Table 1. The phases given in the last column are computed from the time when the brighter component crosses the line of sight and is nearest to the earth, i.e., from  $T + 33.2$  days. Figure 1 shows the velocity-curve corresponding to the final elements. Observations used in the least-squares solution are marked by vertical lines, the half-length of which is equal to the probable error

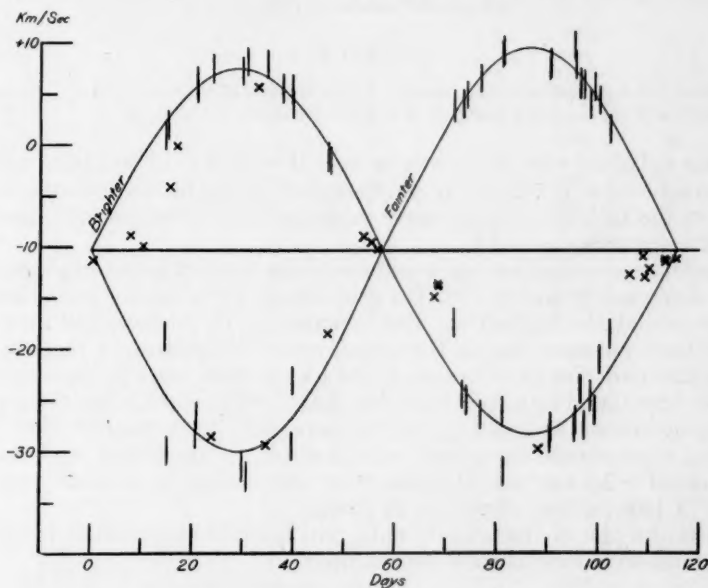


FIG. 1.—Velocity-curve of  $\eta$  Andromedae

of a single observation of unit weight. Observations not used in the solution are marked by crosses. These were: (a) when the two components could not be separated; (b) poor plates; and (c) measures of such low weight that they might be neglected in the solution.

The trigonometric parallax of  $\eta$  Andromedae is given in Schlesinger's *General Catalogue of Stellar Parallaxes* (1935) as  $+0''.001 \pm 0''.006$  and the spectroscopic parallax as  $0''.015$ . On the basis of the spectroscopic parallax and the values given above for  $a_1 \sin i$  and  $a_2 \sin i$ , the maximum angular separation between the two components is  $0''.006$ .

I wish to express sincere thanks to Dr. J. H. Moore for many helpful suggestions in the course of this work.

# THE SPECTROSCOPIC ORBIT OF $\epsilon$ ARIETIS\*

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Received December 4, 1945

## ABSTRACT

The single-lined, G5-type spectroscopic binary  $\epsilon$  Arietis is found to have a period of 1568 days and an orbital eccentricity of 0.36; the half-amplitude of velocity variation is 11 km/sec.

The variable radial velocity of  $\epsilon$  Arietis ( $\alpha = 1^h 51^m 9^s$ ,  $\delta = +17^\circ 20'$ , 1900; vis. mag. = 5.16, G5) was announced in 1922 by W. W. Campbell<sup>1</sup> on the basis of 5 plates. Continued observations at the Lick Observatory have made available 44 plates for a least-squares solution for the spectroscopic orbit.

The spectral lines are single and show no broadening due to a faint component. Table 1 contains the observations and results. The first column gives the Universal Time of observation; the second, the Julian Day. The velocities in the third column were obtained by the writer from measures with a Hartmann spectrocomparator; a plate of  $\alpha$  Boötis was used as a standard. The measures are on the Lick system, since the personal equation had previously been found equal to 0.0 km/sec. An error was noted in one of the published radial velocity determinations, namely, for the plate taken on October 29, 1918 (G.M.T.)<sup>2</sup> The published value should have been  $-3.4$  km/sec, in agreement with the writer's measured value of  $-2.8$  km/sec. All plates were taken with the new Mills three-prism spectrograph ( $\lambda$  4500 central, dispersion 11 Å/mm).

On the basis of a plot of the velocity data, which yielded a period of 1570 days, the following preliminary elements were determined:

$$\begin{array}{ll} T = \text{JD } 2420933 & \mu = 0.229299 \\ \omega = 91^\circ 52' & K = 10.9 \text{ km/sec} \\ e = 0.35 & V = -4.6 \text{ km/sec} \end{array}$$

The observations were grouped into 30 normal places to obtain the differential corrections to the elements by the method of least squares. Observations of only the same epoch were combined, since a correction to the period was to be included among the unknowns. The normal places were weighted according to the number of observations contained in each. The solution resulted in the following final elements, with their probable errors:

$$\begin{array}{ll} T = \text{JD } 2420961.1 \pm 27.2 \text{ days} & K = 10.78 \pm 0.31 \text{ km/sec} \\ \omega = 94^\circ 04' \pm 4^\circ 72' & V = -4.86 \text{ km/sec} \\ e = 0.356 \pm 0.022 & a_1 \sin i = 217 \times 10^6 \text{ km} \\ \mu = 0.229642 \pm 0.000397 & \frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = 0.166 \odot \\ P = 1567.66 \pm 0.62 \text{ days} & \end{array}$$

The probable error of a single observation is  $\pm 0.57$  km/sec.

\* Contributions from the Lick Observatory, Ser. II, No.12.

<sup>1</sup> Pub. A.S.P., 34, 168, 1922.

<sup>2</sup> Pub. Lick Obs., 16, 22, 1928.

TABLE 1  
RADIAL VELOCITIES OF  $\epsilon$  ARIETIS

Date U.T.	Julian Day	Velocity (Km/Sec)	O-C (Km/Sec)	Phase (Days)	Photo- graphed by*
	2420000+				
1918 Oct. 30.....	1896.75	- 2.8	-1.7	935.7	M
1919 Jan. 5.....	1963.62	+ 2.6	+2.2	1002.5	C, P
Nov. 24.....	2286.84	+ 4.3	-1.3	1325.7	C, T
1920 Sept. 11.....	2578.99	-10.2	+0.2	50.2	T
1921 Sept. 29.....	2961.92	-12.7	-0.5	433.1	M
1923 Jan. 15.....	3434.66	- 2.2	-0.5	905.9	Ne
1931 Aug. 20.....	6573.94	- 0.9	+0.7	909.8	P
Nov. 19.....	6664.73	+ 0.7	+0.4	1000.6	P
Nov. 24.....	6669.73	+ 1.0	+0.6	1005.6	M
Nov. 30.....	6675.72	- 0.2	-0.8	1011.6	P
Dec. 16.....	6691.67	+ 1.0	+0.1	1027.6	M
1939 Oct. 15.....	9551.93	- 4.8	+0.2	752.5	Ne
1940 Oct. 7.....	9909.90	+ 2.9	+0.3	1110.5	P
Dec. 10.....	9973.72	+ 4.2	+0.4	1174.3	P
	2430000+				
1941 Sept. 10.....	0247.99	+ 3.8	+0.4	1448.6	M
Nov. 9.....	0307.84	- 1.0	-0.6	1508.4	M, P
Nov. 26.....	0324.77	- 3.3	-1.4	1525.4	P
1942 Jan. 10.....	0369.65	- 5.6	+0.6	2.5	Ne
Jan. 20.....	0379.69	- 7.3	-0.1	12.6	Ne
Mar. 4.....	0422.65	-10.6	+0.3	55.6	Wn
Oct. 16.....	0648.87	-15.3	-0.1	281.8	M, Nt
Oct. 29.....	0661.89	-14.8	+0.2	294.8	Nt, P
Dec. 5.....	0698.77	-14.9	-0.5	331.7	Nt
1943 Jan. 1.....	0725.73	-13.9	-0.1	358.6	M
Jan. 15.....	0739.65	-13.8	-0.2	372.5	Nt
Feb. 15.....	0770.64	-13.5	-0.6	403.5	G
Sept. 10.....	0977.93	- 7.9	+0.3	610.8	P
Sept. 16.....	0983.98	- 9.0	-0.9	616.9	P
Sept. 21.....	0988.01	- 7.8	+0.2	620.9	P
Oct. 10.....	1007.92	- 6.3	+1.2	640.8	P
Nov. 19.....	1029.90	- 5.5	+1.5	662.8	P
Nov. 24.....	1052.84	- 6.0	+0.5	685.7	P
1944 Aug. 29.....	1331.87	+ 0.3	+0.7	964.8	P
Sept. 7.....	1341.00	- 1.1	-0.8	973.9	P
Oct. 1.....	1364.89	- 0.8	-1.1	997.8	P
Oct. 23.....	1386.85	+ 0.9	+0.2	1019.7	P
Nov. 17.....	1411.77	+ 0.7	-0.5	1044.7	P
Dec. 4.....	1428.78	+ 1.7	+0.1	1061.7	P
Dec. 10.....	1434.68	+ 0.7	-1.0	1067.6	P
1945 Jan. 22.....	1477.71	+ 1.4	-1.2	1110.6	P
Aug. 31.....	1698.99	+ 5.6	0.0	1331.9	P
Sept. 23.....	1721.92	+ 6.5	+0.9	1354.8	P
Oct. 20.....	1748.96	+ 5.8	+0.5	1381.9	P
Nov. 14.....	1773.81	+ 6.2	+1.3	1406.7	G, K

\*M = Moore; C = Campbell; P = Paddock; T = Thiele; Ne = Neubauer; Wn = Wirtanen;  
Nt = Nagtegaal; G = Gordon, K = Kron.

The solution reduced the sum of the squares of the residuals from 26.19 to 19.19. Its correctness is shown by the fact that the differences between the residuals derived from the ephemeris and those derived by substitution in the equations of condition do not exceed 0.09 km/sec, except in one instance (0.15 km/sec). The residuals from the ephem-

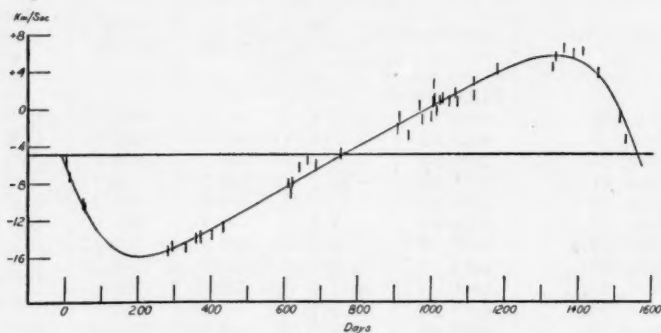


FIG. 1.—Velocity-curve of  $\epsilon$  Arietis

eris are listed in the fourth column of Table 1. Figure 1 shows the velocity-curve corresponding to the final elements. The observations are shown by vertical lines, the half-length of which is equal to the probable error of a single observation. The phases, given in the fifth column of Table 1, are counted from the time of periastron passage.

## THE INFRARED SPECTRUM OF THE MOON\*

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### ABSTRACT

The infrared emission spectra of the moon and earth were observed in 1941 at the Lowell Observatory, Flagstaff, Arizona. These preliminary experiments demonstrate the feasibility of employing the radiations from the moon and earth as tools in the study of the night atmosphere.

This brief paper reports preliminary observations of the infrared spectra of the moon and earth as a means of investigating the night atmosphere. The first and only prior examination of the dispersed lunar spectrum was made by S. P. Langley, with the assist-

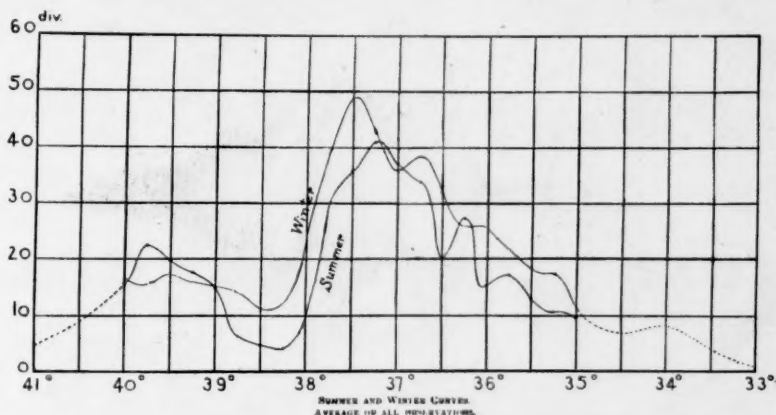


FIG. 1.—The infrared spectrum of the moon (Langley)

ance of F. W. Very.<sup>1</sup> Figure 1 is a graphical summary of Langley's observations of the infrared lunar spectrum. These curves, which are the averages of bolometric measurements made at the Allegheny Observatory during the period 1884-87, are taken from Langley's paper on "The Temperature of the Moon."<sup>1</sup> In the plots of galvanometer deflection versus spectral position, the wave length increases to the right. For purposes of orientation it may be pointed out that the great water band at  $6.3 \mu$  occupies the interval between the deviation settings  $38^\circ$  and  $39^\circ$ . Very<sup>2</sup> has translated the spectrometer circle readings to wave lengths and has found that the spectrum extends beyond  $15.7 \mu$ . This is evidence of contamination of the spectrum by scattered radiations and other sources, for the principal window of atmospheric transmission terminates abruptly at  $13.9 \mu$ .<sup>3</sup> It in nowise detracts from Langley's remarkable accomplishment to indicate the desirability of further observations.

The present work was done with the potassium bromide ( $KBr$ ) prism spectrometer

\* Presented at the Physics Colloquium, University of Michigan, October, 1942.

<sup>1</sup> *Mem. Nat. Acad.*, 4, Part II, 107, 1889.

<sup>2</sup> *U.S. Weather Bur., Bull. G* (W.B. No. 221 [1900]).

<sup>3</sup> Arthur Adel and C. O. Lampland, *Ap. J.*, 91, 481, 1940.

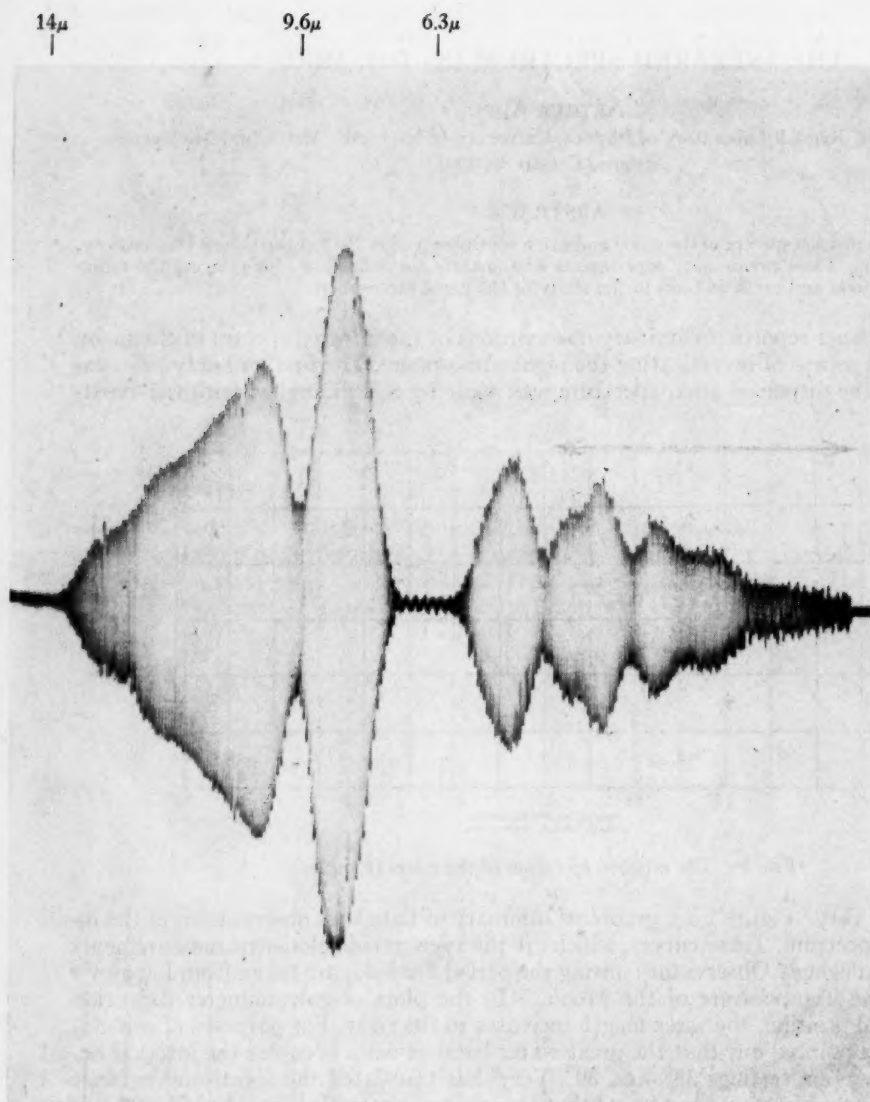


FIG. 2.—The infrared spectrum of the moon. Full moon. Flagstaff, Arizona, September 5, 1941, 10 P.M., M.S.T

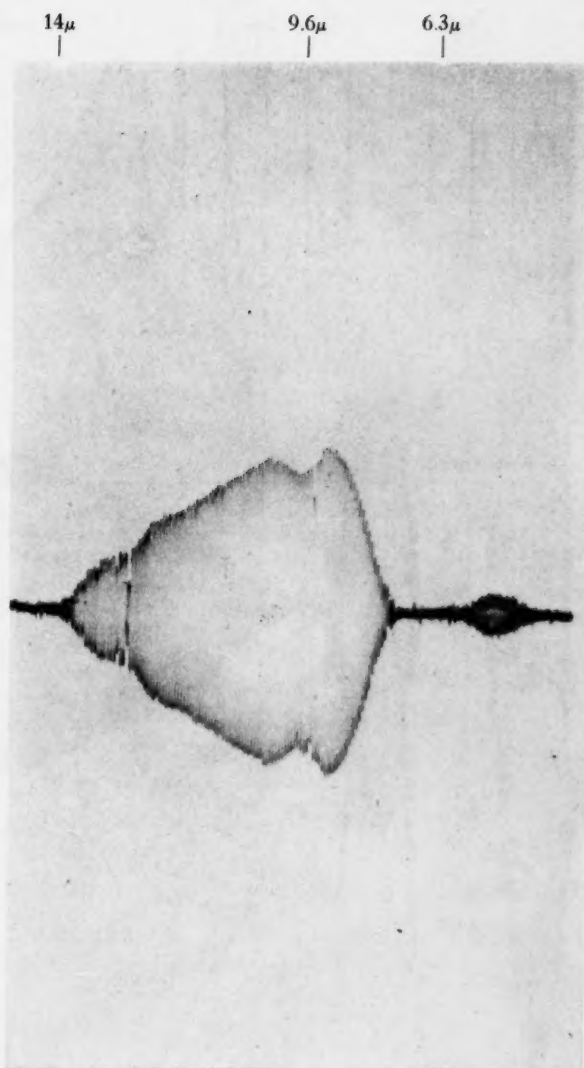


FIG. 3.—The infrared spectrum of the earth. Flagstaff, Arizona, September 5, 1941, 10 P.M., M.S.T.

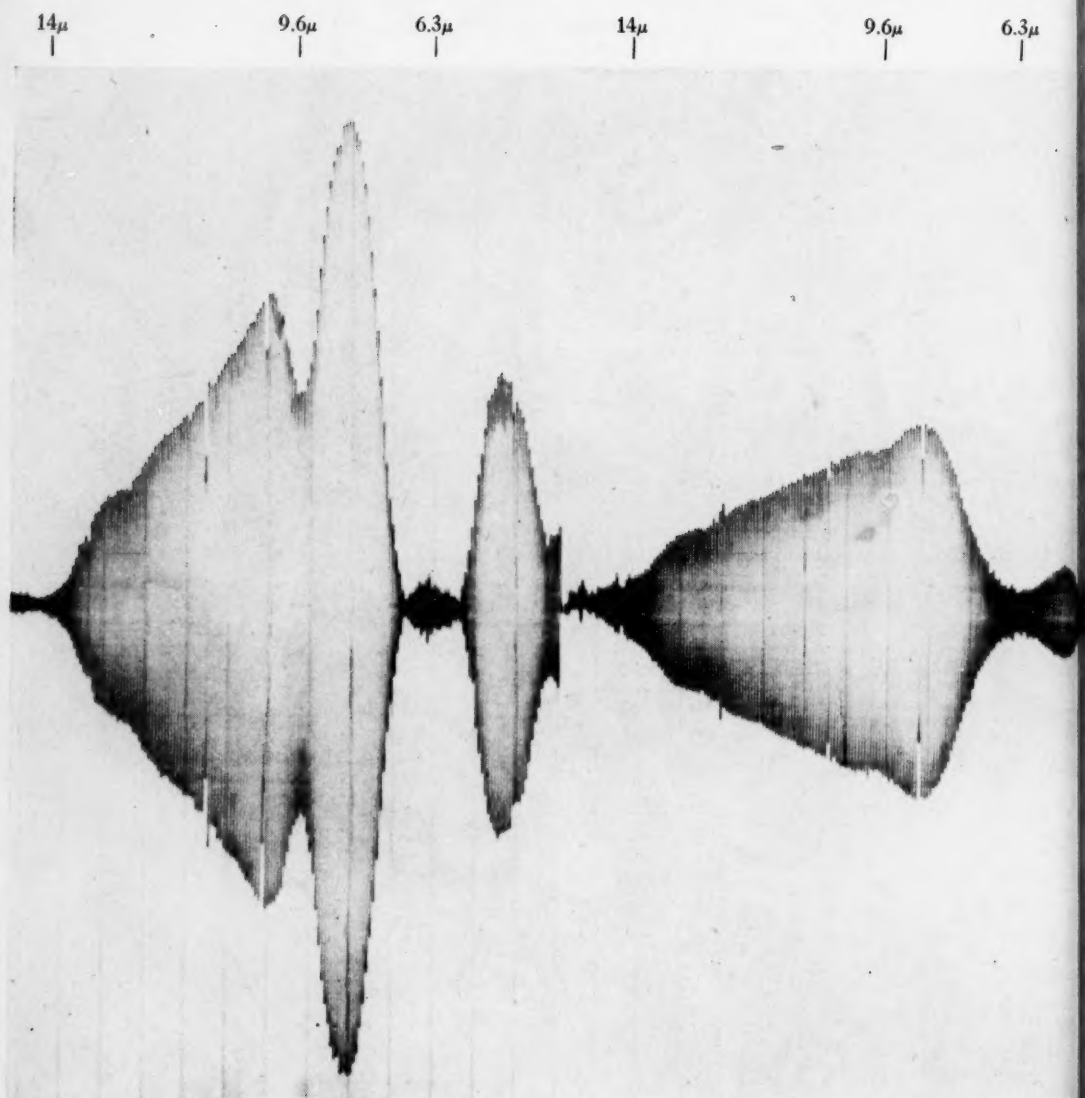


FIG. 4.—Associated infrared spectra of the moon and earth. Full moon. Flagstaff, Arizona, November 3, 1941, 12 P.M., M

6.3 $\mu$   
|



FIG. 5.—The true infrared spectrum of the moon, modified only by the selective absorption of the earth's atmosphere. Obtained by adding the curves of Figures 2 and 3, from which the wave lengths of the principal features may be obtained.

P.M., M

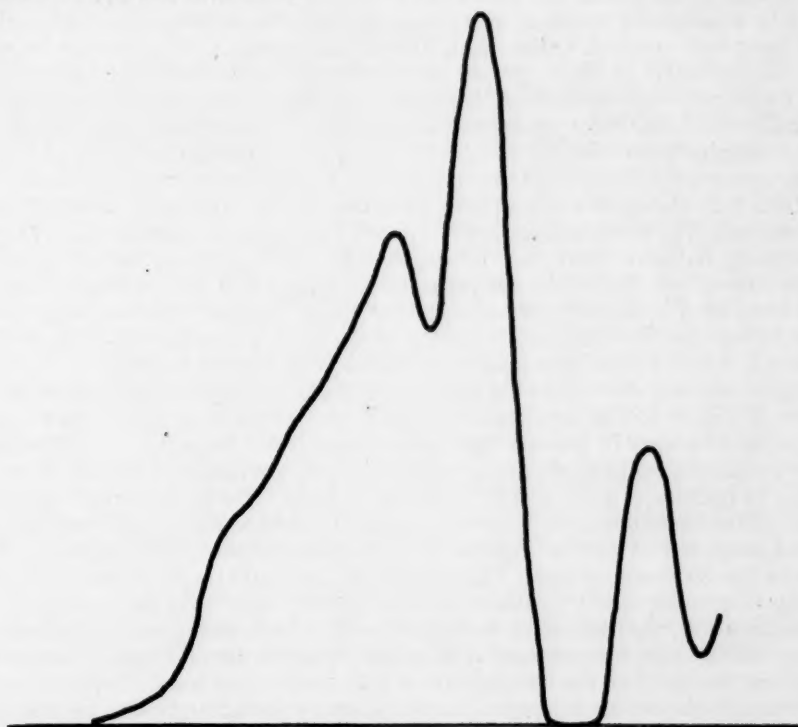


FIG. 6.—The true infrared spectrum of the moon, modified only by the selective absorption of the earth's atmosphere. Obtained by adding the curves of Figure 4.

which had been employed for the extension of the solar spectrum from  $14\ \mu$  to  $24\ \mu$ .<sup>4</sup> Every precaution was taken to eliminate stray radiation, and the lunar and terrestrial spectra, like the extended solar spectrum,<sup>4</sup> are markedly free from contamination.

To obtain the lunar spectrum, the full moon was imaged ( $\frac{3}{4}$  inch diameter) upon the entrance slit-head of the spectrometer, the slit, of width  $\frac{1}{2}$  mm, traversing the entire length (diameter) of the lunar image. The spectrometer was used at aperture  $f/10$ , and the amplification of the galvanometer deflections by no means taxed the signal to noise ratio of the system. With the fast infrared spectrometers now available it should be possible to obtain dispersed infrared emission spectra of detail on the moon, for example, the mountain ranges and the lunar "seas."

The emission spectrum of the earth (thermopile) was secured by permitting the thermopile to radiate to space, via the prism, slits, and coelostat. The recording of a spectrum, either lunar or terrestrial, required but a few minutes; and associated (in time) lunar and terrestrial spectra were recorded in quick succession by having the coelostat retrace its steps with and without the lunar image on the entrance slit of the spectrometer. The thermopile was thus compelled to "look through" the same part of the earth's atmosphere either to the moon or to space. (Of course, lunar and terrestrial emission spectra can be taken during daylight hours, and several have been so obtained.)

Some of the original records of emission by the moon and earth are reproduced in Figures 2, 3, and 4. (It should be pointed out in this connection that the short-wave-length part of Figure 2, indicated by the double arrow, is the spectrum of sunlight as reflected by the moon.) Energy—net received in the case of the lunar spectrum, net radiated by the thermopile in the case of the terrestrial spectrum—is recorded as a function of wave length. The atmospheric bands of water vapor ( $6.3\ \mu$ ), ozone ( $9.6\ \mu$ ), and carbon dioxide ( $13.9\ \mu$ ) have been marked. Other bands are obviously present also. It must be realized that in the formation of these spectra the earth's atmosphere emitted, as well as absorbed, its characteristic radiations. They enter into the terrestrial spectrum as emission bands, and in the lunar spectrum largely as atmospheric absorption bands, but are modified by atmospheric emission. In fact, the lunar spectra depicted in Figures 2 and 4 are the consequences of a complex of events: radiation by the moon, selective absorption of this radiation by the earth's atmosphere, radiation by the thermopile, and radiation by the atmosphere. The terrestrial spectra of Figures 3 and 4 are simpler in origin. They combine outgoing radiation from the thermopile with simultaneously incoming radiation from the atmosphere. Reflection will persuade the reader that the combination, by addition, of associated lunar and terrestrial spectra yields the true lunar emission spectrum as affected only by the selective absorption of the earth's atmosphere. Such reductions of Figures 2, 3, and 4 have been made and are shown in Figures 5 and 6.

The galvanometer deflections are negative in the  $6.3\text{-}\mu$  region of the unreduced lunar spectrum of Figure 4. The significance of this is that at the time of the observation the net exchange of energy in this spectral region was outward from the earth (thermopile), and the situation is to be explained as follows: There is generally sufficient water vapor in the air to operate as a black body in the  $6.3\text{-}\mu$  band. The water vapor thus prevents this part of the lunar spectrum from arriving at the thermopile, which therefore cannot "see" the moon in this spectral interval but merely participates in an exchange of radiations with the water-vapor mass. The thermopile, too, radiates as a black body. Hence, when the thermopile (earth's surface) and the water-vapor layer are essentially at the same temperature, the thermopile receives as much as it radiates, and the galvanometer does not deflect. This was the case at the time the spectrum of Figure 2 was recorded. On the other hand, when the temperature of the water-vapor mass is lower than that of the thermopile, the net exchange is a loss of energy by the latter, producing negative deflections (deflections reversed in sense or phase relative to those produced by incoming radiation). This is the case in Figure 4.

<sup>4</sup> Arthur Adel, *A. P. J.*, 96, 239, 1942.

## RED MAGNITUDES OF THE NORTH POLAR SEQUENCE STARS

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### ABSTRACT

Red magnitudes ranging from 6.25 to 15.35 for 51 stars of the NPS and 17 other stars near the pole were established by means of a neutral filter. A No. 22 Wratten filter and Eastman 103a-E plates were used, yielding an effective wave length of about  $\lambda$  6200. The filter constant was determined by utilizing the blue magnitudes of the stars in the NPS. In spite of all precautions taken to avoid pre-exposure effect, such an effect was present. A method for the determination of this error was developed, and corrections were applied to our results.

The zero point of our magnitude determinations was obtained by assuming

$$\frac{IP_v - Pr}{CI} = K$$

for stars of spectral classes from A0 to A5.

The average probable error of our final magnitudes is  $\pm 0.05$  mag. for the brightest stars,  $\pm 0.03$  mag. for stars of intermediate magnitude, and  $\pm 0.07$  mag. for the faintest stars.

Our results are compared with the red magnitudes determined by the Gaposchkins for the stars common to the two determinations, that is, brighter than red magnitude 13.08. We find no evidence of any over-all scale differences between the two systems, and the average zero-point difference (W and S minus Harvard) is  $+0.06$  mag.

### INTRODUCTION

The purpose of this investigation is to establish standard sequences of red magnitudes to a limiting value of about 15 at the north celestial pole and in the Mount Wilson Selected Areas 39, 40, 64, and 65. The present paper describes the general procedure employed in the determination of red magnitudes and gives the magnitudes for stars of the North Polar Sequence (NPS), together with a few auxiliary stars.

All plates were taken with the 24-36-inch Schmidt-type telescope of the Warner and Swasey Observatory. A No. 22 Wratten filter was used in combination with a neutral filter, which covered half of the photographic plate. Eastman type 103-E, and later 103a-E, plates were used, which, combined with a No. 22 Wratten filter, have an effective wave length of about  $\lambda$  6200. The neutral filter was made by the Eastman Kodak Company, by depositing a coating of white gold on one-half of a circular piece of glass,  $7\frac{3}{4}$  inches in diameter. The thickness of the glass was 0.5 mm. Another piece of glass of the same diameter and thickness was bound with the filter in order to protect its surface. Spectrophotometer tests showed it to be neutral for all practical purposes for wave lengths above  $\lambda$  3660. A year later the filter was tested again in order to determine whether or not its neutral properties had remained the same; no noticeable change had taken place. The two filters were placed in front of the photographic plate, the red filter being placed 1 mm from the plate. The filter also was examined with the Schilt photometer for uniformity throughout its surface, the variations being negligible.

### DETERMINATION OF THE FILTER CONSTANT

The method here considered of determining red magnitudes is based upon the value of the constant magnitude difference resulting from the use of the neutral filter, namely, the filter constant. The importance of a reliable determination for this constant is therefore obvious. In order to determine the filter constant and, at the same time, to establish our general procedure, we used a series of blue plates, Eastman 103a-O, and the photographic magnitudes of the NPS. Our procedure was to take double exposure plates of the north polar region in such a way as to minimize, as far as possible, pre-exposure

effects. One exposure was taken, then the half-filter was rotated  $180^\circ$  in its plane, and a second exposure was made, with the telescope moved a small amount to separate the images. Each plate was left in the telescope for approximately 3 minutes before the first exposure was made. As a further precaution, the plates in general were taken in pairs and the order of rotation reversed. In spite of these precautions, it was found necessary to apply corrections for the pre-exposure effect.

A graduated scale of stellar images and the Schilt-type photometer were available for measuring plates. A comparison was made to determine their relative efficiency and accuracy for this type of photometry. Photographic magnitudes of a number of stars at the north galactic pole, determined from a uniform series of plates, including SA 56 and 57 for calibration, exhibited  $\pm 0.12$  mag. as the probable error of a single determination

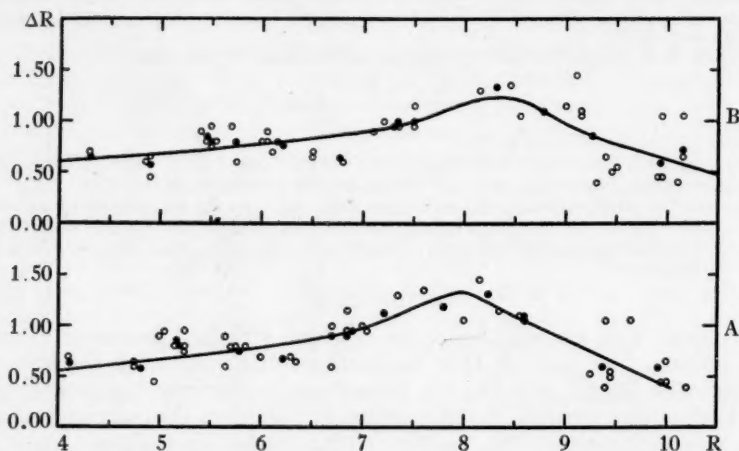


FIG. 1

for both methods. Inasmuch as the Schmidt plates are of excellent quality, especially so for the red plates, the more convenient graduated-image scale was used throughout for the measures.

#### CALIBRATION LINE

Let  $R_B$  represent the graduated scale reading for the bright image of a star, i.e., the nonfilter image, and  $R_F$  the corresponding reading for the faint image of the same star. In order to obtain the value of  $\Delta R = R_F - R_B$  corresponding to any arbitrary value of  $R$ , which is essential in constructing a smooth calibration line, the values of  $\Delta R$  are plotted first against  $R_B$  (Fig. 1, A) and then against  $R_F$  (Fig. 1, B). By combining the results from these two smooth curves, we are able to obtain  $\Delta R$  for any value of  $R$ .

The difference in magnitude between the bright and the faint image of a star is used here as an arbitrary unit. The "magnitudes" expressed in these units are denoted by  $m'$  and have an arbitrary zero point. Figure 2 shows a calibration line formed with these units as abscissas and with the corresponding values of  $R$  as ordinates. This type of curve will be referred to as an " $m'$ -curve."

The insert of Figure 2 shows the method of constructing such a curve. For  $R = 4.0$  we assume  $m' = 0$ ; then  $BA$  is equal to  $\Delta R$  at  $R = 4.0$ , obtained from Figure 1, A and B. Draw  $OA$  until it intersects the line  $R = 4.5$ , that is, point  $C$ . Now take  $CD$  equal to one unit and obtain the value of  $\Delta R$  for  $R = 4.5$  from the same figures; this is the length

DE. Draw CE; CF is the second line segment of our calibration curve. The magnitude of a star may be expressed in the following form:

$$\text{For the nonfilter images, } m_B = \Delta m \cdot m'_B + A, \quad (1)$$

$$\text{For the filter images } m_F = \Delta m (m'_F - 1) + A \quad (2)$$

where  $m_B$  and  $m_F$  denote the magnitudes of a star derived from its bright and faint images respectively,  $\Delta m$  is the filter constant, and  $A$  is the constant for the zero point.

The two equations provide means of obtaining magnitudes when  $\Delta m$  is known, or they may be used in deriving the value of  $\Delta m$  if the magnitudes of the stars are known.

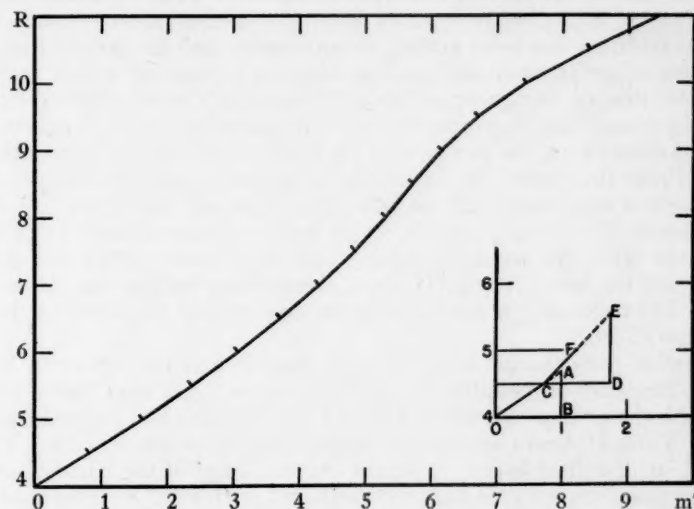


FIG. 2

TABLE 1

Plate No. (1)	$\Delta m$ (2)	$\frac{1}{2} D(\Delta m)_{NPS}$ (3)	$\frac{1}{2} D(\Delta m)_I$ (4)	$\Delta m_c$ (5)	$\Delta m_c$ (6)	$\Delta m_c$ (7)
364.....	0.823	-0.095	-0.112	0.728	0.711	0.813
365.....	.883	- .045	- .097	.838	.786	.815
599.....	.727	+ .002	+ .007	.729	.734	.735
600.....	.850	.087	- .092	.763	.758	.757
606.....	.631	+ .126	+ .154	.757	.785	.803
650.....	.771	+ .030	+ .012	.801	.783	.778
651.....	.749	- .008	- .024	.741	.725	.717
661.....	.764	+ .008	+ .004	.772	.768	.815
662.....	.774	+ .078	+ .044	.852	.818	.789
674.....	.754	+ .030	+ .035	.784	.789	.828
362.....	.737	+ .007	+ .019	.744	.756	.854
363.....	.907	.103	- .076	.804	.831	.722
672.....	.917	- .122	- .084	.795	.833	.833
673.....	0.852	-0.048	-0.024	0.804	0.828	0.791
Average.....				0.779	0.779	0.789

In our case we have used the photographic magnitudes of the stars in the NPS to obtain the value of  $\Delta m$ . Table 1, column (2), gives the uncorrected results obtained from 10 blue-sensitive plates (first ten lines). For comparison purposes, we have used 4 red-sensitive plates with the red filter (last four lines), and for red magnitudes we have used those determined by C. P. Gaposchkin and S. Gaposchkin,<sup>1</sup> which reach the limit 13.27 mag. for star No. 25.

#### PRE-EXPOSURE EFFECT

In spite of the precautions taken to eliminate errors resulting from pre-exposure effects, such errors were present. This became evident as a scale error which showed up in reducing the data described above. Another indication of this effect was the dispersion of the values of  $\Delta m$  for the individual plates, as shown in Table 1, column (2).

These differences in  $\Delta m$  between plates could also be due to variations in uniformity in the neutral filter. As has been stated, its uniformity had previously been tested by direct measures of the transmitted light. As a means of making further tests based on measures of star images, four groups of about 25 stars each were measured on Plate 364, the areas being placed nearly symmetrically with respect to the plate center. A calibration-curve was drawn from the measures of the stars of the NPS and their corresponding magnitudes. From this curve ( $m$ -curve), the magnitudes corresponding to the bright and faint images of each star in all the areas were obtained, and hence the value of  $\Delta m$  was found directly. For Areas I and IV, which were on the same side of the filter line as the stars of the NPS, the resulting mean values were  $\Delta m = 0.914$  and  $\Delta m = 0.964$ , respectively; and for Areas II and III, the corresponding values were  $\Delta m = 0.721$  and  $\Delta m = 0.651$ . The difference in  $\Delta m$  between the two sides of the filter line is attributed to pre-exposure effect.

The amount of pre-exposure effect for this plate is half the difference between the means for the two sides of the filter line, i.e.,  $\frac{1}{2}D(\Delta m) = 0.126$  mag. This correction may be applied with the proper sign to the value of  $\Delta m$  for this plate derived from the NPS stars (col. [2], Table 1), to obtain the true value of the filter constant ( $\Delta m_c$ ). This method was modified, as described below, to permit the reduction of the value of  $\Delta m_c$  without resorting to known magnitudes and was employed with all the plates used in the determination of the filter constant. Two groups, containing about 20 stars each, were measured, one on either side of the filter line, for evaluation of the correction. Instead of using the  $m$ -curve of the NPS stars to obtain directly magnitudes for the auxiliary area stars, we used the  $m'$ -curve of the NPS stars as follows. Let

$m'_b$  = The "magnitude" of the bright image of a star in the units  $m'$ , obtained from the  $m'$ -curve;

$m'_f$  = The corresponding magnitude of the faint image of the star, obtained from the same curve;

$\Delta m' = m'_f - m'_b$ , this should be equal to 1 if errors were not present;

$\overline{\Delta m'_I}$  = The mean value of  $\Delta m'$  for all the area stars on the same side of the filter line as the NPS stars are situated;

$\overline{\Delta m'_{II}}$  = The corresponding mean for the stars on the opposite side.

Then

$$(\overline{\Delta m'_{II}} - \overline{\Delta m'_I}) \cdot \Delta m = D(\Delta m) \quad (3)$$

and

$$\Delta m_c = \Delta m + \frac{1}{2} D(\Delta m). \quad (4)$$

<sup>1</sup> *Harvard Ann.*, 89, 99, Table III, 1935.

The value of  $\Delta m_e$  may be obtained as well, without making use of the  $m'$ -curve of the NPS stars but using in its place an  $m'$ -curve from the area stars on the same side of the filter line as the NPS stars. The procedure is exactly the same, and for the sake of comparison the two methods were used in nearly all our determinations of  $\Delta m_e$ . The agreement between these alternate methods was good, as Table 1 shows. Column (3) was obtained from the  $m'$ -curve of the NPS stars and equation (3), while column (4) was obtained from the  $m'$ -curve of Area I and the same equation. The greatest difference in corresponding entries in the two columns is 0.052 mag., and the least, 0.004 mag. Columns (5) and (6) give the values of  $\Delta m_e$  obtained from the two methods, which, however, are not entirely independent. The mean of  $\Delta m_e$  of the 14 determinations derived by the two methods is exactly the same and is equal to  $0.779 \pm 0.007$  mag.

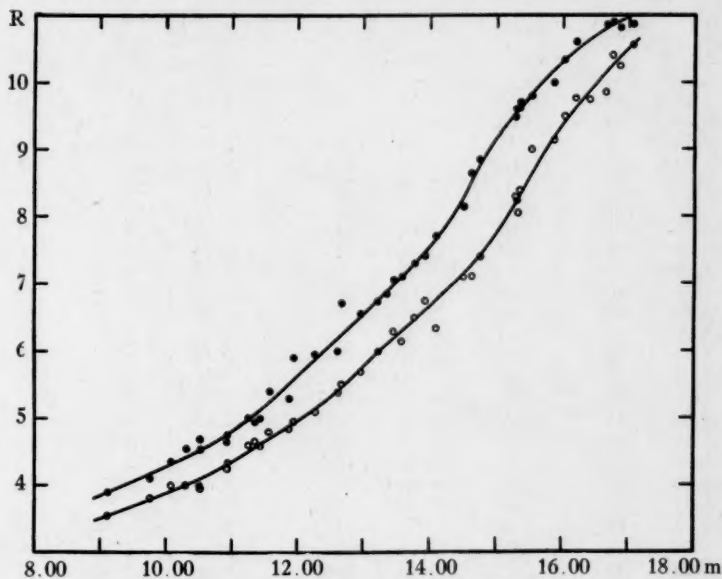


FIG. 3

With the exception of No. 674, the plates were all measured in the same way. Plate 674, however, purposely was taken out of focus, and the measurements were made with the Schilt photometer; the method of reduction was the same.

A check based on an entirely independent method of computing  $\Delta m_e$  was possible by plotting the  $m$ -curve for the nonfilter images of the NPS stars and a similar curve for the filter images. A typical pair of such curves is found in Figure 3. The average difference in abscissa between the two curves gives the  $\Delta m$  for the plate. The resulting values obtained in this manner, corrected for pre-exposure effect listed in column (3), are given in column (7); the mean for 14 plates is  $0.789 \pm 0.008$  mag., which is in good agreement with the value obtained above.

#### DETERMINATION OF RED MAGNITUDES AT THE NORTH CELESTIAL POLE

The determinations of the red magnitudes in the north polar region were made from 18 plates (three groups of 6 each) taken in the manner which we have explained. The exposure time for the first set was 15 seconds; for the second, 60 seconds; and for the third, 5 minutes. Each plate contains two exposures, and the plates were taken in pairs

in order to minimize pre-exposure and other systematic effects. For timing the short exposures, a quick-acting occulting disk was used.

The red magnitudes obtained from the individual plates were found by the general procedure described in connection with the determination of the filter constant. The correction for pre-exposure effect was computed and applied to the adopted value of  $\Delta m_c$  in order to obtain the required  $\Delta m$  for each plate (eq. [4]). Then the magnitudes for each star were derived from equations (1) and (2) by neglecting the  $A$  term. The resulting value—i.e., the mean of  $m_B$  and  $m_F$ —we shall denote as the "plate magnitude," which is, of course, uncorrected for zero point.

TABLE 2

Star	P'r	No. Obs.	p.e.m	Pr	Star	P'r	No. Obs.	p.e.m	Pr
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
5.....	-0.10	10	$\pm 0.05$	6.25	10r.....	5.54	12	$\pm 0.03$	11.89
6.....	+0.44	10	.04	6.79	12r.....	5.68	12	.04	12.03
7.....	1.29	2	.11	7.64	20.....	5.81	12	.04	12.16
4r.....	1.53	12	.02	7.88	21.....	5.88	12	.05	12.23
8.....	1.43	11	.03	7.78	22.....	6.16	12	.03	12.51
5r.....	1.46	7	.05	7.81	23.....	6.30	12	.05	12.65
6r.....	2.26	12	.03	8.61	24.....	6.37	6	.04	12.72
9.....	2.23	12	.04	8.58	25.....	6.73	6	.06	13.08
10.....	2.59	12	.04	8.94	9s.....	6.75	6	.05	13.10
5s.....	3.25	16	.02	9.60	26.....	6.79	6	.05	13.14
4s.....	3.41	18	.02	9.76	8s.....	6.80	6	.05	13.15
11.....	3.29	18	.01	9.64	11s.....	7.40	6	.07	13.75
12.....	3.44	18	.01	9.79	13s.....	7.42	4	.07	13.77
7r.....	3.21	18	.02	9.56	10s.....	7.49	5	.09	13.84
8r.....	3.77	18	.01	10.12	28.....	7.52	6	.07	13.87
13.....	4.02	18	.01	10.37	12s.....	7.65	4	.04	14.00
14.....	4.10	18	.01	10.45	14s.....	7.96	4	.09	14.31
9r.....	4.29	18	.02	10.64	16s.....	8.08	5	.08	14.43
6s.....	4.14	18	.02	10.49	29.....	8.19	6	.08	14.54
15.....	4.45	18	.02	10.80	30.....	8.28	5	.13	14.63
16.....	4.76	15	.02	11.11	31.....	8.31	4	.04	14.66
17.....	4.73	18	.02	11.08	32.....	8.18	4	.09	14.53
11r.....	5.23	12	.02	11.58	15s.....	8.46	4	.11	14.81
7s.....	5.43	10	.03	11.78	33.....	8.76	4	.16	15.11
18.....	5.36	12	.02	11.71	34.....	9.00	4	$\pm 0.04$	15.35
19.....	5.65	12	$\pm 0.03$	12.00					

In order to have a common basis for the comparison of all the plate magnitudes, it was necessary to reduce the determination for each of the 18 plates to a common, arbitrary zero point. For this purpose, 11 stars were chosen, which were measured on all the plates and their mean plate magnitude computed. The average of the differences between these means and the corresponding values of the plate magnitudes for each plate provided a constant for the reduction of the individual plate results to a standard system. The values thus obtained are comparable for all 18 plates, and, except for the application of a zero-point correction, are the final red magnitudes. They are denoted by P'r.

Table 2 summarizes the results for the NPS stars. P'r is given in column (2), followed by the number of determinations used in column (3) and the probable error of the mean of the plate magnitude in column (4). The probable error of a single determination is somewhat larger for the bright and for the faint stars, as might be expected from the increased uncertainty in the estimation of the relatively bright and faint filter images, as well as from the smaller number of observations. If the results are separated into three magnitude groups, the mean probable error of a single determination for the first group

(brightest stars), having 2-12 observations, is  $\pm 0.12$  mag.; for the second, with 12-18 observations,  $\pm 0.09$  mag.; and for the third (faintest stars), with 4-6 observations,  $\pm 0.16$  mag.

## THE ZERO POINT

Seares, Ross, and Joyner<sup>2</sup> in their recent catalogue give the photographic and photovisual magnitudes of all BD stars within  $5^\circ$  of the north celestial pole. Also Seares and Joyner<sup>3</sup> give the intrinsic colors corresponding to the HD spectra and the coefficient of interstellar absorption in the direction of the north pole. From these two sources the zero point of our red magnitudes has been derived. Table 3 gives all the necessary data se-

TABLE 3

Star	IPg	IPv	CI	W and S Sp.	P'r	A	V	Pr
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
5 (88° 4).....	6.49	6.48	+0.01	A0	-0.10	6.58	+21	6.25
6 (89° 13).....	7.18	7.09	+ .09	A2	+0.44	6.64	+27	6.79
7 (88° 64).....	7.36	7.58	- .22	A0	1.29	6.31	- 6	7.64
10 (89° 3).....	9.15	9.07	+ .08	A3	2.59	6.47	+10	8.94
12 (89° 25).....	10.07	9.81	+ .26	A3	3.44	6.35	- 2	9.79
13 (89° 29).....	10.55	10.40	+ .15	A5	4.02	6.37	0	10.37
87° 1.....	9.14	9.17	- .03	A2	2.76	6.41	+ 4	9.11
15.....	8.21	8.18	+ .03	A5	1.91	6.27	-10	8.26
44.....	11.02	10.60	+ .42	A5	4.15	6.42	+ 5	10.50
45.....	9.39	9.25	+ .14	A3	3.13	6.11	-26	9.48
181.....	8.59	8.67	- .08	A2	2.40	6.28	- 9	8.75
205.....	7.46	7.51	- .05	A2	1.22	6.29	- 8	7.57
207.....	11.29	10.80	+ .49	A5	4.43	6.34	- 3	10.78
217.....	8.97	9.07	- .10	A2	2.62	6.46	+ 9	8.97
88° 21.....	11.02	10.74	+ .28	A2	4.34	6.38	+ 1	10.69
39.....	9.28	9.10	+ .18	A2	3.01	6.08	-29	9.36
87.....	11.35	10.88	+ .47	A5	4.47	6.38	+ 1	10.82
106.....	10.51	10.31	+ .20	A3	3.84	6.46	+ 9	10.19
115.....	9.19	9.22	- .03	A3	2.84	6.38	+ 1	9.19
131.....	8.96	8.98	- .02	A3	2.54	6.44	+ 7	8.89
133.....	9.94	9.89	+ .05	A5	3.49	6.40	+ 3	9.84
89° 6.....	10.66	10.31	+ .35	A3	3.83	6.46	+ 9	10.18
38.....	9.88	9.75	+0.13	A0	3.47	6.27	-10	9.82

cured from paper A for stars of spectral classes A0-A5 for which red magnitudes have been determined. The first 6 stars are from the NPS while the rest, 17 in all, are BD stars situated close to the stars of the NPS.

The international photographic magnitudes (IPg) are given in column (2) of Table 3, the corresponding photovisual magnitudes (IPv) in column (3), and the color indices (CI) in column (4). The spectra have been secured from unpublished material of this observatory and are on the HD system. Finally, column (6) gives the values of our red magnitudes (P'r).

The zero point is derived on the assumption that

$$\frac{IPv - Pr}{CI} = K, \quad (5)$$

where

$$Pr = P'r + A. \quad (6)$$

<sup>2</sup> "Magnitudes and Colors of Stars North of  $+80^\circ$ ," *Carnegie Institution of Washington Pub. 532*, 1941; hereafter referred to as "paper A."

<sup>3</sup> *A p. J.*, 98, 279, 1943, hereafter referred to as "paper B."

From a series of approximations, the value of  $K$  was found equal to 0.071. From this and equations (5) and (6), the zero-point correction ( $A$ ) is obtained (col. [7]). The difference between the individual values of  $A$  and the mean value of  $A$  is given in column (8) in units of 0.01 mag.

The entries in Table 3 have been divided into two groups: Group I, which includes all stars brighter than 9<sup>m</sup>25 (IPv), 13 stars in all; and Group II, with 10 stars fainter than 9<sup>m</sup>25 (IPv). The mean value for  $A$  is given in Table 4 and is indicated as "Solution 1."

Paper B shows that a systematic error exists in the color indices for stars of photo-visual magnitude about 9<sup>m</sup>25 and brighter, and spectra earlier than G0. When this correction is applied and the values of IPg and IPv are adjusted to the new color indices, a new value of  $A$  is obtained, and the results are given in Table 4, Solution 2.

There is no significant difference in the zero-point correction determined from Groups I and II or in the two solutions, so we may consider the mean for the 23 stars as a reasonable value for  $A$ .

TABLE 4

SOLUTION	GROUP I		GROUP II		ALL STARS	
	A	p.e.	A	p.e.	A	p.e.
1 . . . . .	6.36	±0.031	6.38	±0.011	6.37	±0.016
2 . . . . .	6.34	.032	6.36	.011	6.35	.017
3 . . . . .	6.36	.032	6.31	.014	6.34	.018
4 . . . . .	6.27	±0.038	6.39	±0.030	6.32	±0.029

An alternate, but not independent, method of obtaining the zero point makes use of the equation:

$$\frac{\text{IPg} - \text{Pr}}{\text{CI}} = K_1. \quad (7)$$

The same procedure was followed as before, the results being shown in Solution 3. The value of  $K_1 = 1.33$  was secured from equation (7) by a series of approximations utilizing the value of P'r from Table 3.

#### THE ZERO POINT DERIVED FROM THE COLOR EXCESS OF THE STARS IN THIS REGION

The exhaustive studies of the intrinsic colors ( $C_s$ ) of all spectral classes reduced to the HD system and the coefficient of selective absorption by Seares and Joyner in paper B are utilized here to secure a check for our zero point.

The values of  $C_s$  for the A-type stars are given in paper B as shown in the accompanying tabulation. The value of  $C_s$  for A0 stars agrees with results obtained from stars in

For A0,	$C_s = -0.15$	For A3,	$C_s = -0.02$
For A2,	$C_s = -0.05$	For A5,	$C_s = 0.00$

unobscured regions. Hence in our solution we may assume that the blue-red color index of spectral class A5 is equal to zero and reduce all spectral classes in our list to A5 by applying as a correction the foregoing values of  $C_s$ .

Table 5 illustrates the zero-point reduction carried out for the same stars that are listed in Table 3. Column (2) gives the uncorrected distance ( $\rho$ ) on the assumption that the absolute magnitudes of paper B are correct. Column (3) gives the color excess ( $E$ ) from the same paper, while column (4) gives the corrected distance ( $\rho_c$ ) obtained from the equation

$$\log \rho_c = \log \rho - 0.75 E.$$

The corrected color excess ( $E_c$ ) is given in column (5), and column (6) reduces each  $E_c$  to that corresponding to an A5-type star, as we have explained.  $IPg - Pr$  (col. 7)), is obtained from column (6) by multiplying the values in the column by  $K_1 = 1.33$ , as computed in the previous solution. Finally  $A$  (col. [8]) was deduced from

$$A = IPg - 1.33 E_{A5} - P'r.$$

The difference between the individual values of  $A$  and the mean value of  $A$  is given in column (9) in units of 0.01 mag.

The stars in Table 5 are divided into the same two groups as before: Group I includes all stars within  $\rho_c = 250$  parsecs, and Group II the rest. The value of the zero point and its probable error are given as Solution 4 in Table 4. The difference in  $A$  between Groups

TABLE 5

Star (1)	$\rho$ (2)	$E$ (3)	$\rho_c$ (4)	$E_c$ (5)	$E_{A5}$ (6)	$1.33 \times E_{A5}$ (7)	$A$ (8)	$V$ (9)
5.....	126	+0.07	112	+0.07	-0.08	-0.11	6.70	+0.38
6.....	126	.07	112	.07	+ .02	+ .03	6.71	+ .39
7.....	209	.09	178	.10	- .05	- .07	6.14	- .18
10.....	295	.14	234	.14	+ .12	+ .16	6.40	+ .08
12.....	417	.20	295	.21	+ .19	+ .25	6.38	+ .06
13.....	550	.24	363	.24	+ .24	+ .32	6.21	- .11
87° 1.....	324	.16	246	.16	+ .11	+ .15	6.23	- .09
15.....	200	.09	170	.09	+ .09	+ .12	6.18	- .14
44.....	603	.25	389	.26	+ .26	+ .35	6.52	+ .20
45.....	324	.16	246	.16	+ .14	+ .19	6.07	- .25
181.....	257	.12	209	.12	+ .07	+ .09	6.10	- .22
205.....	151	.08	132	.08	+ .03	+ .04	6.20	- .12
207.....	661	.26	417	.27	+ .27	+ .36	6.50	+ .18
217.....	309	.15	240	.15	+ .10	+ .13	6.22	- .10
88° 21.....	676	.27	427	.27	+ .22	+ .29	6.39	+ .07
39.....	316	.15	246	.16	+ .11	+ .15	6.12	- .20
87.....	692	.27	436	.27	+ .27	+ .36	6.52	+ .20
106.....	525	.23	355	.24	+ .22	+ .29	6.38	+ .06
115.....	316	.15	246	.16	+ .14	+ .19	6.16	- .16
131.....	288	.13	229	.14	+ .12	+ .16	6.26	- .06
133.....	436	.21	302	.22	+ .22	+ .29	6.16	- .16
89° 6.....	525	.23	355	.24	+ .22	+ .29	6.54	+ .22
38.....	562	+0.24	372	+0.24	+0.09	+0.12	6.29	-0.03

I and II in this solution shows that an overcorrection for absorption might have taken place for the near-by stars, although the magnitude of the probable error for each determination of  $A$  is too large to justify such an assumption.

The first three solutions for  $A$  given above are not independent, but Solution 4 is based on the statistical study of over a thousand stars and can be considered independent of the others. However, the range in the value of  $A$  in all the solutions is only 0.05 mag., so we may conclude that the final value is  $A = 6.35 \pm 0.02$  mag. For the sake of clearly defining our zero point we may state that it was obtained from a group of 23 bright stars with a range in photovisual magnitude from 6.48 to 10.88 and of spectral classes A0-A5, on the assumption that the ratio of the yellow-red indices to the blue-yellow is constant within the narrow limits of spectral type concerned.

## RESULTS

Column (5), Table 2, and column (9), Table 3, give the red magnitudes ( $Pr$ ) of all the stars determined near the north celestial pole; our limiting red magnitude is 15.35. The

probable errors given in column (4), Table 2, refer to  $P'r$  values, the probable errors for  $Pr$  are somewhat greater, owing to the p.e. of the value of  $A$ ; but in no case is the p.e. of  $Pr$  greater than that of  $P'r$  by 0.01 mag.

The residuals from the mean value of  $A$  given in Table 3, column (8), provide a limited check on the scale of our magnitudes. The range in photovisual magnitudes of the stars in this table is from 6.48 to 10.88; and when they are arranged in order of magnitude no systematic trend in the residuals is apparent.

A comparison of our results with those adopted by the Gaposchkins<sup>1</sup> for the stars common in the two determinations, that is, up to red magnitude 13.08, is, of course, of considerable interest, as the observational material is completely independent.

The Gaposchkin magnitudes were determined with the 12-inch Metcalf refractor, ciné red filter, and an Eastman 1-c Special plate, producing an effective wave length of 6300 Å. The scale of magnitude was determined by means of an aluminum objective grating. Their zero point was obtained from stars of spectral classes B9-A2 by so adjusting their colors that the zero yellow-red color index ( $IP_v - Pr$ ) would occur for stars of the same color as in the system ( $IP_g - IP_v$ ).

In comparing the two results we find no evidence of any over-all scale difference. The average zero-point difference (Warner and Swasey *minus* Harvard) in magnitude between the two determinations is +0.06 mag. An alternate determination of the zero point given in the Gaposchkin paper makes their magnitudes 0.13 mag. fainter than their tabulated values. Our results therefore lie exactly between their two determinations.

# SPECTROGRAPHIC OBSERVATIONS OF $\mu$ CEPHEI<sup>1</sup>

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## ABSTRACT

Radial velocities of  $\mu$  Cephei were measured on 116 one-prism spectrograms taken in 1930-1936. Three long cycles (500-1000 days) of the irregular light-variation are covered. The range of velocity in the long cycle is about 5 km/sec. Variations of shorter period and similar range are superimposed. The correlation of light and velocity is similar to that of  $\alpha$  Orionis, with velocity minimum during the decline of light. Weak hydrogen emission appeared at two of the brighter maxima of light.

Variation of the radial velocity of  $\mu$  Cephei was discovered on three-prism spectrograms at the Lick Observatory.<sup>2</sup> When an observing list of red variables was made up at Ann Arbor, this star was placed on a list for later attention. In June, 1930, the writer noticed by chance that the variable was unusually bright. Spectrographic observations were then begun immediately.<sup>3</sup>

## LIGHT-VARIATIONS

Continuous observations by the Argelander method were made, beginning June 8, 1930. The observations up to May 5, 1937, inclusive, were communicated to W. Hassenstein and were used by him in the discussion of the light-curve.<sup>4</sup> Normal places, based on

TABLE 1  
MAXIMA AND MINIMA OF LIGHT

MAXIMA			MINIMA		
JD	Mag.	Notes	JD	Mag.	Notes
2426210.....	3.76	Sharp	2426720:.....	4.1	Very broad
7200.....	3.77		7510.....	4.21	Narrow
7730.....	3.78		8220.....	4.4	
8490.....	4.00				

the writer's estimates, were published in Hassenstein's paper. These are plotted in Figure 1 and on a smaller time-scale in Figure 2. The plots include additional observations up to JD 2429000 (April 11, 1938).<sup>5</sup>

The variation is unpredictably irregular. A long cycle, of several hundred days and range about 0.5 mag., has superimposed waves of the order of 100 days or less and of a range of about 0.1 mag. Table 1 lists maxima and minima of the long cycle for the interval under discussion. The mean period is 756 days. The short cycle is much more difficult to establish. In the formation of means, some of the range of variation probably has been obscured. The interval JD 2426500 to 2427100 shows the short cycle to best advantage.

<sup>1</sup> HD 206,936; (1900)  $\alpha$  21<sup>h</sup>40<sup>m</sup> 4,  $\delta$  +58°19'; mag. 3.6-5.0; Sp. M2.

<sup>2</sup> *Lick Obs. Bull.*, 7, 102, 1912.

<sup>3</sup> McLaughlin, *Pub. A.A.S.*, 6, 369, 1930.

<sup>4</sup> *Potsdam Pub.*, Vol. 29, No. 1, 1938.

<sup>5</sup> The systematic correction -0<sup>m</sup>.06, which Hassenstein applied to the writer's observations, has been included.

## SPECTRAL CHANGES

The first spectrogram, on June 24, 1930, showed hydrogen emission lines at  $H\beta$ ,  $H\gamma$ , and  $H\delta$ , just about equal to the continuous spectrum and filling up the absorption lines. As the star faded, the emission disappeared, and the hydrogen absorption resumed its normal intensity. The intensity ratios  $H\beta: \lambda 4866$  and  $H\gamma: \lambda 4338$  show this effect of the emission lines. Means of these ratios are plotted at the bottom of Figure 2. Emission was evidently at its strongest just before the maximum of JD 2426210. A marked increase of the emission occurred also near the maximum of JD 2427730 but not at the equally bright one about JD 2427200. It is suggested that emission intensity may be more closely correlated with the rate of increase of light than with the brightness attained at maximum.

At the light-maximum of JD 2426210 the  $TiO$  band head  $\lambda 4954$  was practically invisible. As the star faded the band head emerged distinctly. Though later maxima were practically as bright, the band was never again seen so weak as when first observed.

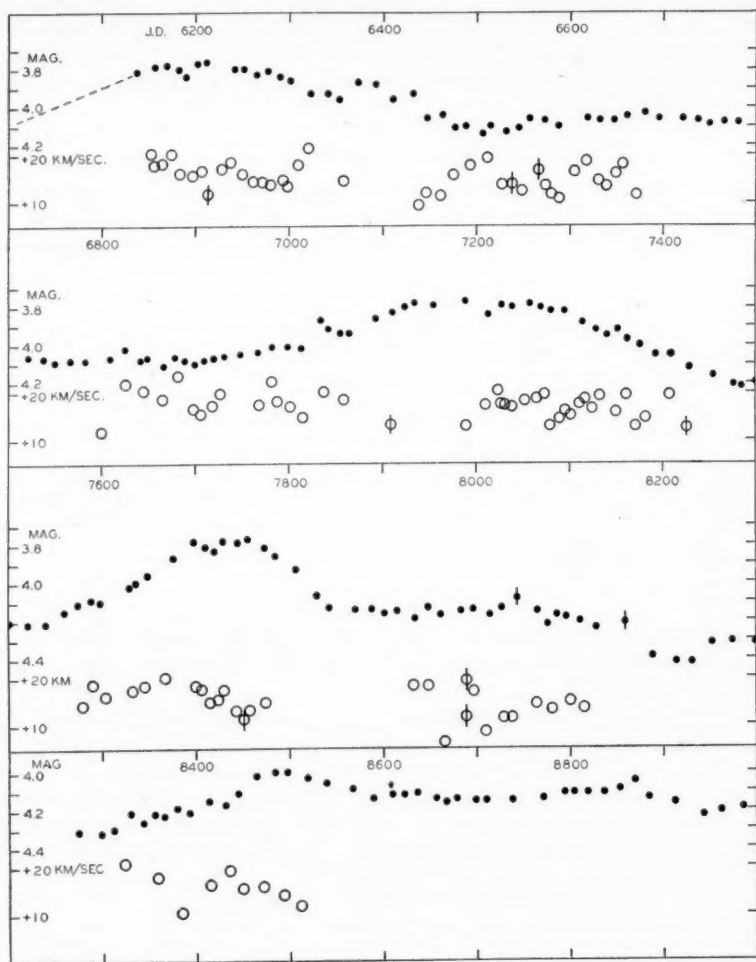


FIG. 1.—Magnitudes and radial velocities of  $\mu$  Cep. Dots are means of several observations of magnitude; open circles, individual radial velocities. Vertically barred symbols indicate low weight.

## RADIAL VELOCITY

Radial velocities measured on 116 Michigan spectrograms are given in Table 2 and plotted in Figure 1. The same star lines and comparison lines were used throughout the measures, and the observations therefore constitute a very homogeneous set. The only exceptions to this occurred when plates were underexposed in the region of shorter wave lengths. However, all lines were examined for systematic deviations, and all appreciable differences were corrected. Hence, the omission of some lines should not produce spurious variations of velocity. The bright lines are not appreciably displaced relative to the absorption spectrum.

The mean radial velocity is  $+16.0$  km/sec. The range is small, and accidental errors may cause considerable distortion of the curves. So far as definitive results are concerned, the data are somewhat disappointing and inconclusive. Nevertheless, distinct trends are present, and some general conclusions appear to be warranted.

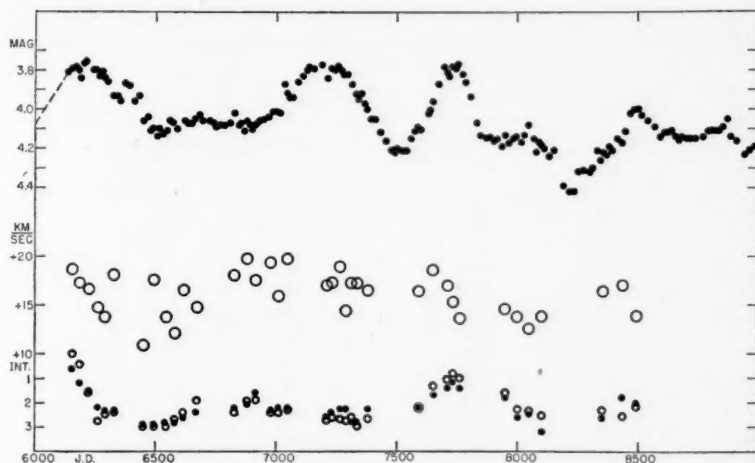


FIG. 2.—Correlated light, velocity, and hydrogen-absorption intensity for  $\mu$  Cep. Upper curve (dots), magnitudes. Middle (open circles), means of radial velocities. Bottom, dots, absorption ratio  $H\beta/\lambda 4866$ ; open circles,  $2 \times H\gamma/\lambda 4338$ .

## DISCUSSION

Figure 1 shows clear evidence of a cyclic variation of velocity in a very irregular interval of a few months. This is similar in length to the shorter cycle of light-variation. During the first two years, when the range of the short cycle was most conspicuous, the correlation of light and velocity was apparently Cepheid-like. In later portions of the series the data are rather inconclusive, owing to the small range of the short-period light-variation.

Correlation of the velocities with the longer period of the light-variation is obscured by the short-period changes in Figure 1. Velocities were therefore grouped into means which partially average out these more rapid changes, and these are plotted in Figure 2. In spite of the remaining scatter, there is clearly a long-period oscillation which can plausibly be correlated with the light-cycle. The range is about 5 km/sec, and the velocity minima occur several months after the light-maxima.<sup>6</sup> The relations of light and velocity thus resemble those shown by  $\alpha$  Orionis.<sup>7</sup>

<sup>6</sup> McLaughlin, *Pub. A.A.S.*, 7, 178, 1933.

<sup>7</sup> Stebbins, *Pub. Washburn Obs.*, 15, 177, 1931.

The relations were investigated further by grouping the observations into means over intervals of one-fourth of a cycle, with the first mean centered on light-maximum. The first cycle thus treated gives the results in Table 3. The trend of the means suggests a lag of velocity minimum somewhat greater than one-fourth of a cycle after light-maximum.

Analyses similar to that of Table 3, coupled with freehand sketched curves on working plots, yielded the dates of velocity maxima and minima given in Table 4. The use of the dates of maxima and minima of light from Table 1 then gives the values of the "lag" in the last column of Table 4.

A comparison of  $\mu$  Cep with  $\alpha$  Ori is of interest. Table 5 gives the principal data. The similarity between these two stars is close enough to warrant the conclusion that they are typical members of the same class of variables, which Ludendorff some years ago named the  $\mu$  Cep class. The most noteworthy point of difference between them is in the amplitude of the shorter light-cycle, which is much smaller in  $\mu$  Cep.

TABLE 2  
RADIAL VELOCITIES OF  $\mu$  CEP

JD	Velocity (Km/Sec)	JD	Velocity (Km/Sec)	JD	Velocity (Km/Sec)
2426152.88	+20.2	2426801.92	+11.7	2427426.57	+12.6*
6155.87	17.8	6825.90	21.9	7578.85	14.2
6164.88	18.1	6845.86	20.3	7590.84	18.9
6173.87	20.1	6866.80	18.7	7603.78	16.1
6183.89	16.0	6881.79	23.6	7632.80	17.4
6197.86	15.5	6898.85	16.7	7645.82	18.4
6206.86	16.5	6906.86	15.4	7667.88	19.9
6213.87	11.7*	6918.85	17.2	7699.79	18.4
6228.76	17.0	6927.74	19.8	7706.77	17.6
6237.81	18.3	6968.81	17.4	7715.72	14.9
6250.78	15.9	6982.71	22.3	7723.67	15.5
6261.75	14.3	6988.73	18.1	7730.64	17.3
6271.68	14.1	7002.70	17.0	7743.62	13.0
6280.75	13.7	7014.68	14.8	7751.59	11.3*
6293.64	14.5	7037.67	20.0	7757.59	13.1
6298.62	13.2	7058.48	18.4	7774.59	14.9
6309.60	17.8	7109.54	13.1*	7932.85	18.4
6320.66	21.2	7189.86	13.3	7948.80	18.2
6357.56	14.4	7210.90	17.4	7965.77	6.3
6438.92	9.4	7224.88	20.4	7988.85	11.8*
6446.87	11.9	7227.77	17.7	7989.86	19.4*
6461.89	11.4	7231.88	17.3	7997.85	17.0
6475.87	15.7	7238.82	17.0	8009.78	8.8
6493.81	17.8	7252.83	18.3	8029.73	11.7
6511.86	19.3	7265.85	18.7	8038.77	11.4
6527.86	13.8	7273.80	19.6	8064.73	14.6
6538.84	13.9*	7279.86	12.9	8080.68	13.2
6548.83	12.3	7288.88	14.3	8100.72	15.0
6566.83	16.9*	7295.78	16.1	8114.72	13.5
6573.86	13.6	7301.79	15.1	8323.81	20.8
6580.83	11.8	7310.79	17.8	8358.71	17.9
6588.82	10.9	7317.80	18.8	8384.84	10.4
6605.74	16.4	7324.84	16.6	8415.71	16.2
6617.76	18.7	7332.79	19.3	8436.70	19.3
6630.62	14.5	7350.59	15.8	8450.67	15.5
6639.69	13.4	7360.73	19.5	8472.75	15.8
6649.72	16.0	7371.66	12.7	8493.60	14.0
6656.68	18.0	7381.68	14.7	8511.52	+11.9
6671.62	+11.6	7407.66	+19.3		

\*Half-weight.

Earlier observations of the radial velocity of  $\mu$  Cep were made at the Lick Observatory<sup>8</sup> and at Victoria.<sup>9</sup> These are listed in Table 6. The first two Lick velocities are remarkably high; no observations in the Michigan series deviate so far from the mean. Hassenstein's light-curve shows a possible reason for this. About JD 2418700 there was an unusually deep minimum of light, magnitude 5.0. The high velocities were observed about 300 days later and probably represent the maximum of velocity associated with the deep light-minimum. The lag is of the same order as that shown by the Michigan observations.

Except for the last observation, the rest of the Lick velocities show a very small range

<sup>8</sup> *Pub. Lick. Obs.*, **16**, 318, 1928.

<sup>9</sup> *Pub. Dom. Ap. Obs.*, **6**, 195, 1934.

TABLE 3  
QUARTER-CYCLE MEANS OF VELOCITY

JD Interval	Light-Phase	Velocity	Number
2426152 to 2426334 .....	Maximum	+16.5	18
6335 to 6582 .....	Decline	14.0	13
6583 to 6829 .....	Minimum	15.3	10
6830 to 7076 .....	Increase	18.6	14
7077 to 7324 .....	Maximum	+16.7	17

TABLE 4  
VELOCITY MAXIMA AND MINIMA

Phase	JD	Velocity (Km/Sec)	Lag (Days)
Minimum .....	2426460	+14.0	250
Maximum .....	6930	18.6	210
Minimum .....	7450?	15.5	250?
Maximum .....	7680	18.5	170
Minimum .....	8000	13.5	270:
Maximum .....	8400:	+16.8	180:

TABLE 5  
COMPARISON OF  $\mu$  CEP AND  $\alpha$  ORI

	$\mu$ Cep	$\alpha$ Ori
Period of short cycle .....	90 $\pm$ days?	200 $\pm$ days
Range of light (short) .....	0.1 mag.	0.3 mag.
Period of long cycle .....	756 days	2100 days
Range of light (long) .....	0.5 mag.	0.5 mag.
Range of velocity .....	5 km/sec	4.1 km/sec
Lag: light-maximum to velocity minimum .....	255 days (0.33 P)	300 days (0.14 P) visual 550 days (0.26 P) photoelectric
Lag: light-minimum to velocity maximum .....	187 days (0.25 P)	300 days (0.14 P) visual 550 days (0.26 P) photoelectric

close to the mean of the Michigan velocities. Again, from Hassenstein's curve, we note that the deep minimum was followed by one cycle of normal range, after which the star settled down to a succession of cycles of unusually small amplitude. It is only to be expected that reduced range of light-variation would be accompanied by smaller changes of velocity. The writer has pointed out such behavior in R. Scuti<sup>10</sup> and R Sagittae.<sup>11</sup>

The Victoria observations and the last one of the Lick series are all high above the Michigan mean and show a rather small range, although the amplitude of light-variation was normal. However, during that interval, JD 2422200 to 2423700, Hassenstein's curve shows that the star's *mean* brightness was far below normal. This anomaly may therefore be reflected in unusually high positive values of the velocity. This phenomenon also was exhibited by R Sge.

TABLE 6  
EARLIER RADIAL VELOCITIES OF  $\mu$  CEP

LICK OBSERVATIONS		VICTORIA OBSERVATIONS	
JD	Velocity (Km/Sec)	JD	Velocity (Km/Sec)
2419005.6 . . . . .	+29.4	2422506.9 . . . . .	+24.7
9035.6 . . . . .	27.1	2568.8 . . . . .	25.1
9262.8 . . . . .	15	2633.6 . . . . .	28.0
9265.8 . . . . .	14.8	2887.9 . . . . .	24.2
9268.8 . . . . .	15.6	3021.6 . . . . .	23.1
9273.8 . . . . .	16.4	3295.8 . . . . .	22.4
9274.9 . . . . .	19.0	3337.8 . . . . .	22.1
9592.9 . . . . .	19.0	3704.7 . . . . .	+22.7
9598.9 . . . . .	19.4		
19634.0 . . . . .	18.9		
22228.7 . . . . .	+25.0		

The available data all form a consistent picture, and there appears to be no reason to doubt the general correctness of the ranges of velocity and the approximate phase relations shown by the Michigan one-prism observations. On the other hand, the quantities measured are close to the limit of reliable measurement, and there is little to attract investigators to the detailed study of other similar stars with such dispersion. Co-ordinated photometric and three-prism spectrographic observations of  $\mu$  Cep over a few cycles would perhaps be worth while, but even more valuable information would be obtained from a similar study of  $\alpha$  Ori.

Reduction of the measures of radial velocity was materially assisted by a grant from the Alexander Dallas Bache Fund of the National Academy of Sciences, which is gratefully acknowledged.

<sup>10</sup> *Pub. Michigan Obs.*, 7, 69, 1938.

<sup>11</sup> *A p. J.*, 94, 97, 1941.

# THE MOTION OF AN ELECTRON IN THE HARTREE FIELD OF A HYDROGEN ATOM

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## ABSTRACT

The radial wave functions  $\chi_0$  and  $\chi_1$  (of unit amplitude at infinity) of an electron moving in the static field of a hydrogen atom in its ground state and having an angular momentum of respectively 0 and 1 Bohr unit are tabulated for a range of values of the kinetic energy which is of astrophysical interest. Certain auxiliary quantities, such as the phase shifts, are also tabulated.

**1. Introduction.**—The wave functions of an electron in the static field of a hydrogen atom are essential for the solution of several problems of astrophysical interest. Perhaps the most important of these problems is the theoretical evaluation of the continuous absorption coefficient of the negative hydrogen ion.<sup>1</sup> However, the basic physical problem, namely, that of the motion of an electron in the field of a hydrogen atom, is well known in the quantum theory and has been considered particularly in investigations relating to the phenomenon of electron scattering.<sup>2</sup> In this latter context it has been studied by H. S. W. Massey and C. B. O. Mohr,<sup>3</sup> J. McDougall,<sup>4</sup> and P. M. Morse and W. P. Allis.<sup>5</sup> From the investigations of these writers it would appear that in an approximation in which exchange is ignored the "best wave function,"  $\Psi$ , which is separable in the co-ordinates of the two electrons describing the hydrogen atom, together with an electron moving with a definite velocity at infinity, is expressible in the form

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{\pi}} e^{-r_1} \phi(\mathbf{r}_2; \mathbf{k}), \quad (1)$$

where, adopting Hartree's atomic units,  $e^{-r_1}/\sqrt{\pi}$  represents the wave function of the hydrogen atom in its ground state and  $\phi(\mathbf{r}; \mathbf{k})$  is the wave function of an electron having a momentum  $\mathbf{k}$ , in the Hartree field

$$-\left(1 + \frac{1}{r}\right)e^{-2r} \quad (2)$$

of the hydrogen atom;  $\phi(\mathbf{r}; \mathbf{k})$  therefore satisfies the wave equation

$$\nabla^2 \phi + \left[ k^2 + 2 \left( 1 + \frac{1}{r} \right) e^{-2r} \right] \phi = 0. \quad (3)$$

<sup>1</sup> Cf. S. Chandrasekhar, *Ap. J.*, **102**, 395, 1945, where the integrations tabulated in this paper have already been used to compute the absorption cross-sections of  $H^-$  arising from the bound-free transitions. Similar calculations for the free-free transitions will be found in a forthcoming paper.

<sup>2</sup> Cf. N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions*, Oxford University Press, 1933.

<sup>3</sup> *Proc. R. Soc., A*, **136**, 289, 1932.

<sup>4</sup> *Proc. R. Soc., A*, **136**, 549, 1932.

<sup>5</sup> *Phys. Rev.*, **44**, 269, 1933.

Expanding  $\phi$  in zonal harmonics in the form

$$\phi = \sum_{l=0}^{\infty} r \chi_l P_l(\cos \vartheta) \quad (4)$$

the radial functions  $\chi_l$  satisfy the equation

$$\frac{d^2 \chi_l}{dr^2} + \left[ k^2 - \frac{l(l+1)}{r^2} + 2 \left( 1 + \frac{1}{r} \right) e^{-2r} \right] \chi_l = 0. \quad (5)$$

For the purposes of the evaluation of the absorption cross-sections radial functions are required which tend to pure sinusoidal waves of unit amplitude at infinity. More particularly, solutions are required whose behavior at infinity is of the form

$$\chi_l \rightarrow \sin(kr - \frac{1}{2}l\pi + \delta_l) \quad (r \rightarrow \infty), \quad (6)$$

where the  $\delta_l$ 's are the so-called "phase shifts." It may be recalled in this connection that, according to the theory of Faxén and Holtsmark,<sup>6</sup> the total elastic cross-section  $Q$  of the atom for electrons with momentum  $k$  is given by

$$Q = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2 \delta_l \quad (7)$$

which accounts for the theoretical importance of the phase shifts.

For most purposes, however, the s- and the p-waves  $\chi_0$  and  $\chi_1$  are the most important; and we shall accordingly provide in this paper tables of these solutions for a range of values of  $k^2$  which is of astrophysical interest.\*

2. *The s-waves*  $\chi_0$ .—The s-radial wave functions  $\chi_0$  satisfy the equation

$$\frac{d^2 \chi_0}{dr^2} + \left[ k^2 + 2 \left( 1 + \frac{1}{r} \right) e^{-2r} \right] \chi_0 = 0, \quad (8)$$

and solutions of this equation are required whose behavior at infinity is

$$\chi_0(r; k) \rightarrow \sin(kr + \delta_0) \quad (r \rightarrow \infty). \quad (9)$$

Such solutions can be found in the following manner:

There exists a solution  $X_0$  of equation (8) whose behavior at the origin is given by

$$X_0(r; k^2) = r - r^2 + \frac{1}{6}(4 - k^2)r^3 - \frac{1}{8}(5 - 2k^2)r^4 + \dots \quad (10)$$

With a series expansion of this form valid for  $r \rightarrow 0$  and including thirteen terms it is possible to determine  $X_0$  to a sufficient accuracy for  $r < 0.5$ . For  $r \geq 0.5$  the solution can be continued by any of the standard methods of numerical integration. For  $r > 8$

\* Cf. *op. cit.*, p. 24.

the term in  $e^{-2r}$  in equation (8) is entirely negligible. The solution must, accordingly, be of the form

$$X_0 = A_0 \sin(kr + \delta_0) \quad (r > 8), \quad (11)$$

where  $A_0$  and  $\delta_0$  are certain determinate constants.

The phase shift  $\delta_0$  can be found from the formula (readily established)

$$\tan \delta_0 = \frac{X_0(r_1) \sin kr_2 - X_0(r_2) \sin kr_1}{-X_0(r_1) \cos kr_2 + X_0(r_2) \cos kr_1} \quad (r_1, r_2 > 8). \quad (12)$$

Once  $\delta_0$  has been determined from values of  $X_0$  for two different values of  $r > 8$ ,  $A_0$  immediately follows, and the s-wave  $\chi_0$  is given by

$$\chi_0 = \frac{1}{A_0} X_0. \quad (13)$$

In this manner the radial functions  $\chi_0$  have been found for thirty different values of  $k^2$  in the range  $1.75 \geq k^2 \geq 0.015$ . The integrations were carried out keeping seven significant figures; and the final solutions, rounded to five decimals, are given in Table 5.

In Table 1 we have collected the values of the phase shifts  $\delta_0$ . It will be seen that, even for  $k^2 = 0.015$ , the phase shift is quite large.

TABLE 1  
THE PHASE SHIFT  $\delta_0$

$k^2$	$\delta_0$	$k^2$	$\delta_0$	$k^2$	$\delta_0$
1.75.....	0.82323	0.20.....	1.05361	0.040.....	0.97313
1.50.....	0.84675	.175.....	1.05652	.035.....	.95215
1.25.....	0.87379	.150.....	1.05796	.030.....	.93196
1.00.....	0.90567	.125.....	1.05673	.025.....	.90206
0.90.....	0.91995	.100.....	1.05041	.020.....	.86322
0.80.....	0.93563	.090.....	1.04582	.015.....	.80769
0.70.....	0.95270	.080.....	1.03919	.010.....	.730
0.60.....	0.97098	.070.....	1.03010	.005.....	.574
0.50.....	0.99092	.060.....	1.01736	.0025.....	.434
0.45.....	1.00145	.055.....	1.00891	0.0010.....	0.288
0.35.....	1.02336	.050.....	0.99932		
0.25.....	1.04479	0.045.....	0.98736		

For  $k^2 < 0.015$  the solutions can be found by a "perturbation" method. For, writing  $X_0(r; k^2)$  in the form

$$X_0(r; k^2) = X_0(r; 0) - k^2 Y_0(r) + O(k^4), \quad (14)$$

it can readily be shown that

$$Y_0(r) = X_0(r; 0) \int_0^r \frac{d\xi}{X_0^2(\xi; 0)} \int_0^\xi d\eta X_0^2(\eta; 0). \quad (15)$$

In Table 2 we have tabulated the functions  $X_0(r; 0)$  and  $Y_0(r)$ . Direct comparison with the integration for  $k^2 = 0.015$  shows that solution (14) agrees with the results of the exact integration for  $r < 6.0$  to within one part in five hundred. It would accordingly

appear that for  $k^2 < 0.015$  equation (14) and our tabulation of the functions  $X_0(r; 0)$  and  $Y_0(r)$  can be used to determine  $\chi_0$  to an accuracy of the order of one part in a thousand. The values of the phase shifts for  $k^2 \leq 0.010$  given in Table 1 were found from the radial functions determined in this manner.

3. *The p-waves*  $\chi_1$ .—The p-radial functions satisfy the equation

$$\frac{d^2 \chi_1}{dr^2} + \left[ k^2 - \frac{2}{r^2} + 2 \left( 1 + \frac{1}{r} \right) e^{-2r} \right] \chi_1 = 0, \quad (16)$$

TABLE 2  
THE FUNCTIONS  $X_0(r; 0)$  AND  $Y_0(r)$

$r$	$X_0(r; 0)$	$Y_0(r)$	$r$	$X_0(r; 0)$	$Y_0(r)$
0.	0	0	3.1.	0.59768	1.67711
0.1.	0.09064	0.00016	3.2.	.60259	1.81160
0.2.	.16490	0.00117	3.3.	.60748	1.95204
0.3.	.22585	0.00373	3.4.	.61235	2.09848
0.4.	.27603	0.00837	3.5.	.61720	2.25099
0.5.	.31748	0.01549	3.6.	.62203	2.40961
0.6.	.35191	0.02544	3.7.	.62686	2.57441
0.7.	.38066	0.03849	3.8.	.63167	2.74544
0.8.	.40483	0.05489	3.9.	.63648	2.92275
0.9.	.42531	0.07484	4.0.	.64127	3.10639
1.0.	.44281	0.09852	4.1.	.64607	3.29641
1.1.	.45790	0.12609	4.2.	.65086	3.49288
1.2.	.47105	0.15771	4.3.	.65564	3.69584
1.3.	.48263	0.19351	4.4.	.66042	3.90534
1.4.	.49293	0.23362	4.5.	.66520	4.12142
1.5.	.50220	0.27818	4.6.	.66998	4.34415
1.6.	.51064	0.32730	4.7.	.67476	4.57357
1.7.	.51839	0.38109	4.8.	.67953	4.80972
1.8.	.52559	0.43967	4.9.	.68431	5.05267
1.9.	.53235	0.50312	5.0.	.68908	5.30244
2.0.	.53874	0.57155	5.1.	.69385	5.55911
2.1.	.54483	0.64506	5.2.	.69862	5.82270
2.2.	.55068	0.72372	5.3.	.70339	6.09327
2.3.	.55634	0.80764	5.4.	.70816	6.37088
2.4.	.56183	0.89689	5.5.	.71293	6.65556
2.5.	.56719	0.99154	5.6.	.71770	6.94738
2.6.	.57245	1.09168	5.7.	.72247	7.24637
2.7.	.57761	1.19738	5.8.	.72724	7.55258
2.8.	.58271	1.30870	5.9.	.73201	7.86608
2.9.	.58774	1.42572	6.0.	0.73678	8.18689
3.0.	0.59273	1.54850			

and solutions of this equation are required whose behavior at infinity is

$$\chi_1(r; k) \rightarrow -\cos(kr + \delta_1) \quad (r \rightarrow \infty). \quad (17)$$

Such solutions can be found in the following manner:

There exists a solution  $X_1$  of equation (13) whose behavior at the origin is given by

$$X_1(r; k^2) = r^2 - \frac{1}{2} r^3 + \frac{1}{10} (3 - k^2) r^4 - \frac{1}{180} (16 - 7k^2) r^5 + \dots \quad (18)$$

Again with a series expansion of this form including thirteen terms it is possible to find  $X_1$  to a sufficient accuracy for  $r < 0.5$ ; the solution can be continued beyond this point by numerical methods. For  $r > 8$  the term in  $e^{-2r}$  in equation (16) can be ignored, and the solution must accordingly be of the form

$$X_1 = A_1 \left[ \frac{\sin(kr + \delta_1)}{kr} - \cos(kr + \delta_1) \right] \quad (r > 8). \quad (19)$$

The phase shift  $\delta_1$  can be determined from the formula (readily established)

$$\tan \delta_1 = \frac{r_1 X_1(r_1) (1 + k^2 r_2^2)^{\frac{1}{2}} \sin(kr_2 - q_2) - r_2 X_1(r_2) (1 + k^2 r_1^2)^{\frac{1}{2}} \sin(kr_1 - q_1)}{r_2 X_1(r_2) (1 + k^2 r_1^2)^{\frac{1}{2}} \cos(kr_1 - q_1) - r_1 X_1(r_1) (1 + k^2 r_2^2)^{\frac{1}{2}} \cos(kr_2 - q_2)}, \quad (20)$$

where

$$q_1 = \tan^{-1} k r_1 \quad \text{and} \quad q_2 = \tan^{-1} k r_2. \quad (21)$$

Once  $\delta_1$  has been determined according to equation (20), using the values of  $X_1$  for two different values of  $r > 8.0$ ,  $A_1$  directly follows, and the p-wave  $\chi_1$  becomes determinate. In this manner the radial functions  $\chi_1$  have been found for twenty-nine different values of  $k^2$  in the range  $1.75 \geq k^2 \geq 0.015$ . The integrations have been carried out to the same accuracy as the s-waves, and the final solutions (also rounded to five decimals) are given in Table 6.

In Table 3 we have collected the values of the phase shifts  $\delta_1$ .

TABLE 3  
THE PHASE SHIFT  $\delta_1$

$k^2$	$\delta_1$	$k^2$	$\delta_1$	$k^2$	$\delta_1$
1.75.....	0.158696	0.25.....	0.026045	0.055.....	0.003291
1.50.....	.148549	.20.....	.019580	.050.....	.002870
1.25.....	.131742	.175.....	.016462	.045.....	.002464
1.00.....	.111510	.150.....	.013398	.040.....	.002075
0.80.....	.092438	.125.....	.010456	.035.....	.001710
0.70.....	.081824	.100.....	.007689	.030.....	.001363
0.60.....	.070441	.090.....	.006632	.025.....	.001042
0.50.....	.058380	.080.....	.005623	.020.....	.000739
0.45.....	.052097	.070.....	.004644	0.015.....	0.000489
0.35.....	0.039178	0.060.....	0.003732		

As in the case of the s-waves, for  $k^2 < 0.015$  the solutions for the p-waves can be found to an accuracy of about one part in a thousand from the formula

$$X_1(r; k^2) = X_1(r; 0) - k^2 Y_1(r), \quad (22)$$

where

$$Y_1(r) = X_1(r; 0) \int_0^r \frac{d\xi}{X_1^2(\xi; 0)} \int_0^\xi d\eta X_1^2(\eta; 0). \quad (23)$$

Equation (22) agrees with the results of the exact integration for  $k^2 = 0.015$  to within one part in a thousand for  $r \leq 6$ . The functions  $X_1(r; 0)$  and  $Y_1(r)$  are tabulated in Table 4.

4. *Concluding remarks.*—As stated in the Introduction, the s- and the p-waves tabulated in this paper have been used for the evaluation of the continuous absorption coefficient of  $H^-$ . It should, however, be admitted that, while this represents an improvement over earlier work, the importance of exchange, particularly for the slow s-electrons, may lead to further changes in the cross-sections for the free-free transitions. We hope to return to these and related matters in the near future.

TABLE 4  
THE FUNCTIONS  $X_1(r; 0)$  AND  $Y_1(r)$

$r$	$X_1(r; 0)$	$Y_1(r)$	$r$	$X_1(r; 0)$	$Y_1(r)$
0	0	0	3.1	5.83783	5.95478
0.1	0.00953	0.00001	3.2	6.20841	6.73891
0.2	0.03645	0.00015	3.3	6.59083	7.59800
0.3	0.07871	0.00073	3.4	6.98513	8.53669
0.4	0.13475	0.00223	3.5	7.39132	9.55977
0.5	0.20341	0.00529	3.6	7.80939	10.67215
0.6	0.28389	0.01070	3.7	8.23937	11.8789
0.7	0.37559	0.01940	3.8	8.68126	13.1852
0.8	0.47813	0.03241	3.9	9.13506	14.5965
0.9	0.59128	0.05094	4.0	9.60078	16.1182
1.0	0.71491	0.07630	4.1	10.07842	17.7558
1.1	0.84898	0.10992	4.2	10.5680	19.5153
1.2	0.99349	0.15340	4.3	11.0695	21.4025
1.3	1.14849	0.20845	4.4	11.5829	23.4235
1.4	1.31407	0.27691	4.5	12.1082	25.5844
1.5	1.49032	0.36077	4.6	12.6454	27.8916
1.6	1.67734	0.46216	4.7	13.1946	30.3516
1.7	1.87525	0.58333	4.8	13.7557	32.9709
1.8	2.08414	0.72668	4.9	14.3287	35.7564
1.9	2.30413	0.89475	5.0	14.9137	38.7149
2.0	2.53530	1.09022	5.1	15.5105	41.8534
2.1	2.77777	1.31591	5.2	16.1193	45.1793
2.2	3.03159	1.57477	5.3	16.7399	48.6997
2.3	3.29687	1.86990	5.4	17.3725	52.4222
2.4	3.57365	2.20455	5.5	18.0170	56.3544
2.5	3.86200	2.58208	5.6	18.6734	60.5039
2.6	4.16199	3.00602	5.7	19.3417	64.8788
2.7	4.47365	3.48002	5.8	20.0218	69.4871
2.8	4.79704	4.00789	5.9	20.7139	74.3368
2.9	5.13217	4.59357	6.0	21.4179	79.4364
3.0	5.47910	5.24112			

TABLE 5

THE RADIAL S-WAVE FUNCTIONS  $\chi_0$  OF AN ELECTRON  
IN THE HARTREE FIELD OF A HYDROGEN ATOM

$r$	$k^2 = 1.75$	$k^2 = 1.50$	$k^2 = 1.25$	$k^2 = 1.00$	$k^2 = 0.90$	$k^2 = 0.80$
0	0	0	0	0	0	0
0.1	+0.26172	+0.25343	+0.24454	+0.23491	+0.23085	+0.22655
0.2	+0.47166	+0.45734	+0.44191	+0.42508	+0.41795	+0.41039
0.3	+0.63534	+0.61754	+0.59812	+0.57672	+0.56758	+0.55785
0.4	+0.75764	+0.73905	+0.71835	+0.69509	+0.68504	+0.67424
0.5	+0.84281	+0.82617	+0.80695	+0.78458	+0.77472	+0.76397
0.6	+0.89459	+0.88262	+0.86760	+0.84886	+0.84028	+0.83067
0.7	+0.91632	+0.91162	+0.90344	+0.89098	+0.88473	+0.87733
0.8	+0.91102	+0.91601	+0.91713	+0.91347	+0.91059	+0.90643
0.9	+0.88149	+0.89831	+0.91101	+0.91851	+0.91995	+0.92002
1.0	+0.83037	+0.86086	+0.88712	+0.90794	+0.91460	+0.91980
1.1	+0.76022	+0.80582	+0.84733	+0.88335	+0.89604	+0.90723
1.2	+0.67355	+0.73528	+0.79335	+0.84618	+0.86562	+0.88355
1.3	+0.57286	+0.65126	+0.72681	+0.79773	+0.82452	+0.84985
1.4	+0.46064	+0.55577	+0.64927	+0.73920	+0.77383	+0.80712
1.5	+0.33939	+0.45077	+0.56226	+0.67174	+0.71456	+0.75624
1.6	+0.21165	+0.33827	+0.46729	+0.59644	+0.64766	+0.69805
1.7	+0.07991	+0.22024	+0.36586	+0.51438	+0.57408	+0.63335
1.8	-0.05330	+0.09868	+0.25947	+0.42663	+0.49472	+0.56291
1.9	-0.18555	-0.02445	+0.14961	+0.33426	+0.41049	+0.48749
2.0	-0.31443	-0.14720	+0.03778	+0.23831	+0.32229	+0.40784
2.1	-0.43764	-0.26767	-0.07454	+0.13985	+0.23100	+0.32469
2.2	-0.55302	-0.38401	-0.18590	+0.03992	+0.13754	+0.23881
2.3	-0.65853	-0.49446	-0.29488	-0.06042	+0.04279	+0.15094
2.4	-0.75235	-0.59735	-0.40009	-0.16014	-0.05237	+0.06181
2.5	-0.83284	-0.69117	-0.50021	-0.25823	-0.14704	-0.02783
2.6	-0.89862	-0.77449	-0.59399	-0.35369	-0.24036	-0.11724
2.7	-0.94857	-0.84610	-0.68026	-0.44556	-0.33148	-0.20569
2.8	-0.98182	-0.90493	-0.75795	-0.53292	-0.41957	-0.29248
2.9	-0.99781	-0.95010	-0.82610	-0.61490	-0.50386	-0.37689
3.0	-0.99629	-0.98097	-0.88387	-0.69069	-0.58357	-0.45827
3.1	-0.97729	-0.99708	-0.93054	-0.75953	-0.65799	-0.53595
3.2	-0.94116	-0.99819	-0.96554	-0.82074	-0.72646	-0.60931
3.3	-0.88854	-0.98430	-0.98844	-0.87372	-0.78836	-0.67778
3.4	-0.82037	-0.95564	-0.99896	-0.91793	-0.84315	-0.74081
3.5	-0.73783	-0.91263	-0.99698	-0.95294	-0.89033	-0.79789
3.6	-0.64239	-0.85592	-0.98253	-0.97841	-0.92948	-0.84857
3.7	-0.53570	-0.78638	-0.95579	-0.99408	-0.96026	-0.89246
3.8	-0.41965	-0.70504	-0.91710	-0.99980	-0.98239	-0.92919
3.9	-0.29626	-0.61313	-0.86694	-0.99552	-0.99567	-0.95849
4.0	-0.16769	-0.51203	-0.80595	-0.98128	-0.99998	-0.98011
4.1	-0.03618	-0.40326	-0.73489	-0.95722	-0.99530	-0.99389
4.2	+0.09595	-0.28844	-0.65465	-0.92360	-0.98165	-0.99972
4.3	+0.22641	-0.16930	-0.56622	-0.88074	-0.95917	-0.99754
4.4	+0.35291	-0.04762	-0.47073	-0.82908	-0.92807	-0.98739
4.5	+0.47324	+0.07477	-0.36935	-0.76913	-0.88861	-0.96934
4.6	+0.58530	+0.19605	-0.26336	-0.70149	-0.84116	-0.94354
4.7	+0.68714	+0.31438	-0.15408	-0.62685	-0.78614	-0.91020
4.8	+0.77696	+0.42801	-0.04287	-0.54594	-0.72406	-0.86957
4.9	+0.85321	+0.53522	+0.06887	-0.45957	-0.65546	-0.82199
5.0	+0.91455	+0.63441	+0.17975	-0.36861	-0.58096	-0.76784

TABLE 5—Continued

$r$	$k^2 = 1.75$	$k^2 = 1.50$	$k^2 = 1.25$	$k^2 = 1.00$	$k^2 = 0.90$	$k^2 = 0.80$
5.1.....	+0.95990	+0.72409	+0.28838	-0.27397	-0.50124	-0.70755
5.2.....	+0.98848	+0.80293	+0.39342	-0.17659	-0.41701	-0.64160
5.3.....	+0.99979	+0.86974	+0.49354	-0.07744	-0.32903	-0.57052
5.4.....	+0.99363	+0.92352	+0.58750	+0.02248	-0.23810	-0.49488
5.5.....	+0.97011	+0.96346	+0.67413	+0.12218	-0.14502	-0.41528
5.6.....	+0.92963	+0.98896	+0.75234	+0.22065	-0.05064	-0.33236
5.7.....	+0.87291	+0.99966	+0.82115	+0.31692	+0.04420	-0.24679
5.8.....	+0.80093	+0.99537	+0.87971	+0.41002	+0.13864	-0.15923
5.9.....	+0.71496	+0.97617	+0.92728	+0.49903	+0.23183	-0.07041
6.0.....	+0.61649	+0.94235	+0.96327	+0.58305	+0.32294	+0.01898
6.1.....	+0.50725	+0.89441	+0.98724	+0.66124	+0.41114	+0.10821
6.2.....	+0.38915	+0.83308	+0.99888	+0.73283	+0.49565	+0.19658
6.3.....	+0.26424	+0.75926	+0.99804	+0.79709	+0.57569	+0.28338
6.4.....	+0.13472	+0.67406	+0.98474	+0.85339	+0.65056	+0.36791
6.5.....	+0.00284	+0.57877	+0.95915	+0.90116	+0.71958	+0.44950
6.6.....	-0.12909	+0.47481	+0.92158	+0.93993	+0.78212	+0.52750
6.7.....	-0.25876	+0.36373	+0.87250	+0.96931	+0.83763	+0.60128
6.8.....	-0.38390	+0.24721	+0.81252	+0.98900	+0.88561	+0.67025
6.9.....	-0.50235	+0.12698	+0.74240	+0.99882	+0.92563	+0.73386
7.0.....	-0.61201	+0.00484	+0.66301	+0.99865	+0.95732	+0.79161
7.1.....	-0.71098	-0.11736	+0.57535	+0.98850	+0.98040	+0.84302
7.2.....	-0.79752	-0.23781	+0.48049	+0.96848	+0.99466	+0.88770
7.3.....	-0.87013	-0.35470	+0.37964	+0.93878	+0.99998	+0.92528
7.4.....	-0.92753	-0.46627	+0.27405	+0.89970	+0.99630	+0.95546
7.5.....	-0.96872	-0.57085	+0.16503	+0.85163	+0.98366	+0.97800
7.6.....	-0.99299	-0.66688	+0.05396	+0.79505	+0.96218	+0.99273
7.7.....	-0.99991	-0.75292	-0.05779	+0.73053	+0.93204	+0.99952
7.8.....	-0.98935	-0.82768	-0.16882	+0.65871	+0.89352	+0.99831
7.9.....	-0.96151	-0.89004	-0.27774	+0.58030	+0.84696	+0.98913
8.0.....	-0.91686	-0.93907	-0.38319	+0.49610	+0.79279	+0.97203
8.1.....	-0.85620	-0.97403	-0.48385	+0.40694	+0.73149	+0.94717
8.2.....	-0.78057	-0.99439	-0.57847	+0.31371	+0.66361	+0.91473
8.3.....	-0.69130	-0.99986	-0.66586	+0.21735	+0.58976	+0.87499
8.4.....	-0.58995	-0.99035	-0.74495	+0.11882	+0.51061	+0.82824
8.5.....	-0.47829	-0.96600	-0.81473	+0.01910	+0.42686	+0.77487
8.6.....	-0.35828	-0.92718	-0.87433	-0.08081	+0.33928	+0.71531
8.7.....	-0.23200	-0.87447	-0.92302	-0.17991	+0.24865	+0.65004
8.8.....	-0.10166	-0.80865	-0.96018	-0.27722	+0.15578	+0.57956
8.9.....	+0.03044	-0.73073	-0.98535	-0.37175	+0.06151	+0.50445
9.0.....	+0.16202	-0.64185	-0.99822	-0.46257	-0.03332	+0.42531
9.1.....	+0.29076	-0.54336	-0.99862	-0.54877	-0.12784	+0.34277
9.2.....	+0.41443	-0.43673	-0.98656	-0.62949	-0.22122	+0.25749
9.3.....	+0.53085	-0.32356	-0.96217	-0.70392	-0.31260	+0.17015
9.4.....	+0.63799	-0.20554	-0.92577	-0.77131	-0.40117	+0.08145
9.5.....	+0.73399	-0.08444	-0.87781	-0.83100	-0.48614	-0.00790
9.6.....	+0.81716	+0.03793	-0.81888	-0.88238	-0.56673	-0.09719
9.7.....	+0.88605	+0.15972	-0.74973	-0.92495	-0.64223	-0.18570
9.8.....	+0.93946	+0.27912	-0.67122	-0.95827	-0.71195	-0.27273
9.9.....	+0.97645	+0.39434	-0.58432	-0.98202	-0.77527	-0.35758
10.0.....	+0.99638	+0.50364	-0.49013	-0.99596	-0.83162	-0.43957

TABLE 5—Continued

$r$	$k^2 = 0.70$	$k^2 = 0.60$	$k^2 = 0.50$	$k^2 = 0.45$	$k^2 = 0.35$	$k^2 = 0.25$
0. ....	0	0	0	0	0	0
0.1. ....	+0.22206	+0.21739	+0.21241	+0.20979	+0.20419	+0.19791
0.2. ....	+0.40248	+0.39423	+0.38540	+0.38075	+0.37079	+0.35958
0.3. ....	+0.54761	+0.53690	+0.52537	+0.51927	+0.50617	+0.49134
0.4. ....	+0.66279	+0.65073	+0.63765	+0.63068	+0.61562	+0.59842
0.5. ....	+0.75242	+0.74012	+0.72661	+0.71935	+0.70350	+0.68511
0.6. ....	+0.82012	+0.80868	+0.79585	+0.78886	+0.77333	+0.75491
0.7. ....	+0.86886	+0.85935	+0.84827	+0.84209	+0.82798	+0.81065
0.8. ....	+0.90107	+0.89454	+0.88627	+0.88141	+0.86978	+0.85462
0.9. ....	+0.91876	+0.91620	+0.91173	+0.90873	+0.90060	+0.88866
1.0. ....	+0.92359	+0.92595	+0.92626	+0.92559	+0.92196	+0.91425
1.1. ....	+0.91693	+0.92511	+0.93113	+0.93326	+0.93509	+0.93259
1.2. ....	+0.89996	+0.91480	+0.92738	+0.93277	+0.94097	+0.94463
1.3. ....	+0.87369	+0.89594	+0.91590	+0.92496	+0.94041	+0.95116
1.4. ....	+0.83899	+0.86934	+0.89743	+0.91055	+0.93408	+0.95280
1.5. ....	+0.79667	+0.83571	+0.87259	+0.89013	+0.92253	+0.95006
1.6. ....	+0.74745	+0.79567	+0.84193	+0.86423	+0.90621	+0.94337
1.7. ....	+0.69200	+0.74980	+0.80594	+0.83329	+0.88551	+0.93306
1.8. ....	+0.63096	+0.69863	+0.76507	+0.79772	+0.86078	+0.91943
1.9. ....	+0.56498	+0.64268	+0.71971	+0.75787	+0.83229	+0.90272
2.0. ....	+0.49465	+0.58243	+0.67027	+0.71410	+0.80033	+0.88313
2.1. ....	+0.42059	+0.51837	+0.61711	+0.66672	+0.76512	+0.86085
2.2. ....	+0.34339	+0.45096	+0.56059	+0.61604	+0.72690	+0.83603
2.3. ....	+0.26367	+0.38069	+0.50106	+0.56237	+0.68587	+0.80882
2.4. ....	+0.18203	+0.30803	+0.43888	+0.50601	+0.64224	+0.77936
2.5. ....	+0.09907	+0.23344	+0.37441	+0.44725	+0.59621	+0.74776
2.6. ....	+0.01540	+0.15741	+0.30800	+0.38639	+0.54798	+0.71416
2.7. ....	-0.06839	+0.08042	+0.23999	+0.32374	+0.49775	+0.67866
2.8. ....	-0.15168	+0.00293	+0.17076	+0.25959	+0.44572	+0.64138
2.9. ....	-0.23390	-0.07458	+0.10066	+0.19425	+0.39208	+0.60243
3.0. ....	-0.31447	-0.15164	+0.03005	+0.12801	+0.33704	+0.56193
3.1. ....	-0.39281	-0.22777	-0.04072	+0.06120	+0.28080	+0.51999
3.2. ....	-0.46838	-0.30253	-0.11128	-0.00590	+0.22356	+0.47671
3.3. ....	-0.54066	-0.37546	-0.18128	-0.07296	+0.16552	+0.43223
3.4. ....	-0.60914	-0.44613	-0.25036	-0.13970	+0.10690	+0.38665
3.5. ....	-0.67333	-0.51411	-0.31819	-0.20580	+0.04791	+0.34009
3.6. ....	-0.73280	-0.57899	-0.38442	-0.27098	-0.01125	+0.29267
3.7. ....	-0.78713	-0.64039	-0.44872	-0.33492	-0.07038	+0.24452
3.8. ....	-0.83594	-0.69793	-0.51077	-0.39736	-0.12925	+0.19575
3.9. ....	-0.87889	-0.75129	-0.57026	-0.45800	-0.18768	+0.14648
4.0. ....	-0.91568	-0.80013	-0.62690	-0.51658	-0.24544	+0.09685
4.1. ....	-0.94606	-0.84416	-0.68040	-0.57282	-0.30235	+0.04698
4.2. ....	-0.96981	-0.88313	-0.73049	-0.62649	-0.35819	-0.00301
4.3. ....	-0.98677	-0.91680	-0.77693	-0.67734	-0.41278	-0.05299
4.4. ....	-0.99682	-0.94496	-0.81948	-0.72513	-0.46592	-0.10284
4.5. ....	-0.99990	-0.96745	-0.85793	-0.76966	-0.51743	-0.15243
4.6. ....	-0.99598	-0.98414	-0.89210	-0.81073	-0.56713	-0.20163
4.7. ....	-0.98508	-0.99492	-0.92180	-0.84815	-0.61484	-0.25034
4.8. ....	-0.96730	-0.99973	-0.94689	-0.88175	-0.66040	-0.29841
4.9. ....	-0.94274	-0.99855	-0.96725	-0.91138	-0.70364	-0.34574
5.0. ....	-0.91159	-0.99138	-0.98277	-0.93691	-0.74442	-0.39221

TABLE 5—Continued

$r$	$k^2 = 0.70$	$k^2 = 0.60$	$k^2 = 0.50$	$k^2 = 0.45$	$k^2 = 0.35$	$k^2 = 0.25$
5.1.....	-0.87406	-0.97825	-0.99338	-0.95823	-0.78260	-0.43769
5.2.....	-0.83041	-0.95927	-0.99902	-0.97524	-0.81804	-0.48208
5.3.....	-0.78096	-0.93452	-0.99967	-0.98786	-0.85061	-0.52526
5.4.....	-0.72604	-0.90418	-0.99533	-0.99603	-0.88021	-0.56713
5.5.....	-0.66604	-0.86841	-0.98600	-0.99972	-0.90672	-0.60759
5.6.....	-0.60137	-0.82743	-0.97176	-0.99892	-0.93007	-0.64652
5.7.....	-0.53251	-0.78149	-0.95265	-0.99362	-0.95015	-0.68384
5.8.....	-0.45991	-0.70387	-0.92878	-0.98385	-0.96691	-0.71944
5.9.....	-0.38410	-0.67586	-0.90027	-0.96965	-0.98029	-0.75325
6.0.....	-0.30560	-0.61680	-0.86726	-0.95109	-0.99024	-0.78518
6.1.....	-0.22496	-0.55403	-0.82992	-0.92826	-0.99672	-0.81514
6.2.....	-0.14274	-0.48795	-0.78843	-0.90124	-0.99972	-0.84307
6.3.....	-0.05953	-0.41894	-0.74300	-0.87018	-0.99922	-0.86889
6.4.....	+0.02410	-0.34741	-0.69385	-0.83520	-0.99522	-0.89254
6.5.....	+0.10756	-0.27380	-0.64124	-0.79646	-0.98774	-0.91395
6.6.....	+0.19027	-0.19855	-0.58542	-0.75413	-0.97680	-0.93308
6.7.....	+0.27164	-0.12211	-0.52668	-0.70842	-0.96244	-0.94988
6.8.....	+0.35112	-0.04494	-0.46530	-0.65952	-0.94472	-0.96431
6.9.....	+0.42814	+0.03250	-0.40160	-0.60765	-0.92369	-0.97632
7.0.....	+0.50216	+0.10975	-0.33589	-0.55304	-0.89943	-0.98590
7.1.....	+0.57268	+0.18634	-0.26849	-0.49595	-0.87202	-0.99301
7.2.....	+0.63918	+0.26181	-0.19976	-0.43663	-0.84156	-0.99764
7.3.....	+0.70122	+0.33571	-0.13003	-0.37535	-0.80816	-0.99977
7.4.....	+0.75834	+0.40760	-0.05965	-0.31237	-0.77193	-0.99941
7.5.....	+0.81017	+0.47704	+0.01103	-0.24799	-0.73300	-0.99654
7.6.....	+0.85632	+0.54362	+0.08165	-0.18250	-0.69150	-0.99119
7.7.....	+0.89649	+0.60694	+0.15187	-0.11618	-0.64759	-0.98336
7.8.....	+0.93038	+0.66662	+0.22132	-0.04934	-0.60140	-0.97307
7.9.....	+0.95776	+0.72230	+0.28967	+0.01772	-0.55312	-0.96035
8.0.....	+0.97844	+0.77365	+0.35657	+0.08471	-0.50290	-0.94523
8.1.....	+0.99228	+0.82036	+0.42169	+0.15131	-0.45091	-0.92774
8.2.....	+0.99917	+0.86216	+0.48469	+0.21723	-0.39735	-0.90794
8.3.....	+0.99907	+0.89878	+0.54528	+0.28217	-0.34240	-0.88586
8.4.....	+0.99198	+0.93001	+0.60314	+0.34585	-0.28625	-0.86157
8.5.....	+0.97796	+0.95566	+0.65799	+0.40797	-0.22910	-0.83513
8.6.....	+0.95709	+0.97559	+0.70955	+0.46825	-0.17115	-0.80660
8.7.....	+0.92953	+0.98966	+0.75756	+0.52642	-0.11259	-0.77605
8.8.....	+0.89546	+0.99779	+0.80179	+0.58223	-0.05364	-0.74356
8.9.....	+0.85513	+0.99995	+0.84200	+0.63542	+0.00549	-0.70922
9.0.....	+0.80882	+0.99610	+0.87801	+0.68576	+0.06461	-0.67310
9.1.....	+0.75684	+0.98629	+0.90964	+0.73300	+0.12350	-0.63529
9.2.....	+0.69958	+0.97055	+0.93671	+0.77695	+0.18195	-0.59590
9.3.....	+0.63741	+0.94900	+0.95911	+0.81740	+0.23977	-0.55502
9.4.....	+0.57079	+0.92176	+0.97671	+0.85418	+0.29676	-0.51275
9.5.....	+p. 50018	+0.88899	+0.98943	+0.88711	+0.35270	-0.46920
9.6.....	+0.42606	+0.85088	+0.99720	+0.91606	+0.40741	-0.42448
9.7.....	+0.34897	+0.80768	+0.99999	+0.94088	+0.46069	-0.37870
9.8.....	+0.26943	+0.75963	+0.99779	+0.96147	+0.51237	-0.33197
9.9.....	+0.18801	+0.70703	+0.99059	+0.97773	+0.56225	-0.28441
10.0.....	+0.10527	+0.65019	+0.97845	+0.98960	+0.61016	-0.23613

TABLE 5—Continued

$r$	$k^2 = 0.20$	$k^2 = 0.175$	$k^2 = 0.150$	$k^2 = 0.125$	$k^2 = 0.10$	$k^2 = 0.09$
0	0	0	0	0	0	0
0.1	+0.19430	+0.19225	+0.19006	+0.18751	+0.18443	+0.18299
0.2	+0.35311	+0.34944	+0.34549	+0.34090	+0.33535	+0.33276
0.3	+0.48271	+0.47782	+0.47253	+0.46636	+0.45888	+0.45537
0.4	+0.58832	+0.58255	+0.57631	+0.56898	+0.56004	+0.55584
0.5	+0.67418	+0.66788	+0.66103	+0.65292	+0.64297	+0.63826
0.6	+0.74375	+0.73724	+0.73011	+0.72158	+0.71100	+0.70596
0.7	+0.79985	+0.79342	+0.78633	+0.77770	+0.76686	+0.76165
0.8	+0.84472	+0.83868	+0.83191	+0.82351	+0.81273	+0.80749
0.9	+0.88021	+0.87482	+0.86866	+0.86077	+0.85038	+0.84524
1.0	+0.90776	+0.90329	+0.89802	+0.89092	+0.88122	+0.87631
1.1	+0.92857	+0.92528	+0.92115	+0.91512	+0.90638	+0.90183
1.2	+0.94358	+0.94172	+0.93898	+0.93429	+0.92679	+0.92270
1.3	+0.95354	+0.95337	+0.95227	+0.94917	+0.94318	+0.93967
1.4	+0.95909	+0.96083	+0.96162	+0.96036	+0.95614	+0.95330
1.5	+0.96071	+0.96460	+0.96752	+0.96834	+0.96614	+0.96409
1.6	+0.95881	+0.96507	+0.97035	+0.97349	+0.97356	+0.97240
1.7	+0.95371	+0.96258	+0.97044	+0.97613	+0.97872	+0.97854
1.8	+0.94569	+0.95737	+0.96804	+0.97652	+0.98185	+0.98276
1.9	+0.93498	+0.94967	+0.96336	+0.97485	+0.98317	+0.98526
2.0	+0.92175	+0.93966	+0.95657	+0.97129	+0.98283	+0.98619
2.1	+0.90617	+0.92748	+0.94782	+0.96598	+0.98096	+0.98570
2.2	+0.88838	+0.91327	+0.93723	+0.95903	+0.97768	+0.98388
2.3	+0.86849	+0.89714	+0.92490	+0.95054	+0.97307	+0.98082
2.4	+0.84661	+0.87917	+0.91091	+0.94059	+0.96720	+0.97659
2.5	+0.82285	+0.85946	+0.89534	+0.92924	+0.96014	+0.97126
2.6	+0.79728	+0.83809	+0.87826	+0.91655	+0.95193	+0.96486
2.7	+0.76999	+0.81512	+0.85972	+0.90258	+0.94263	+0.95746
2.8	+0.74107	+0.79062	+0.83979	+0.88737	+0.93227	+0.94907
2.9	+0.71059	+0.76465	+0.81852	+0.87096	+0.92089	+0.93973
3.0	+0.67863	+0.73729	+0.79595	+0.85339	+0.90850	+0.92946
3.1	+0.64526	+0.70859	+0.77214	+0.83469	+0.89515	+0.91830
3.2	+0.61058	+0.67861	+0.74712	+0.81491	+0.88086	+0.90627
3.3	+0.57464	+0.64741	+0.72096	+0.79407	+0.86564	+0.89338
3.4	+0.53753	+0.61505	+0.69368	+0.77221	+0.84953	+0.87965
3.5	+0.49934	+0.58160	+0.66535	+0.74937	+0.83255	+0.86510
3.6	+0.46013	+0.54712	+0.63600	+0.72557	+0.81471	+0.84975
3.7	+0.41999	+0.51168	+0.60568	+0.70085	+0.79604	+0.83363
3.8	+0.37901	+0.47533	+0.57445	+0.67524	+0.77657	+0.81674
3.9	+0.33726	+0.43814	+0.54235	+0.64878	+0.75631	+0.79910
4.0	+0.29484	+0.40017	+0.50942	+0.62150	+0.73528	+0.78074
4.1	+0.25182	+0.36151	+0.47573	+0.59344	+0.71351	+0.76167
4.2	+0.20830	+0.32221	+0.44132	+0.56463	+0.69102	+0.74190
4.3	+0.16436	+0.28234	+0.40625	+0.53512	+0.66785	+0.72147
4.4	+0.12009	+0.24198	+0.37057	+0.50493	+0.64399	+0.70038
4.5	+0.07558	+0.20119	+0.33433	+0.47411	+0.61949	+0.67866
4.6	+0.03092	+0.16005	+0.29758	+0.44270	+0.59438	+0.65633
4.7	-0.01381	+0.11863	+0.26039	+0.41073	+0.56866	+0.63340
4.8	-0.05850	+0.07700	+0.22281	+0.37824	+0.54237	+0.60990
4.9	-0.10308	+0.03523	+0.18489	+0.34529	+0.51555	+0.58586
5.0	-0.14746	-0.00659	+0.14670	+0.31190	+0.48820	+0.56128

TABLE 5—Continued

$r$	$k^2 = 0.20$	$k^2 = 0.175$	$k^2 = 0.150$	$k^2 = 0.125$	$k^2 = 0.10$	$k^2 = 0.09$
5.1.....	-0.19153	-0.04841	+0.10829	+0.27812	+0.46037	+0.53620
5.2.....	-0.23523	-0.09014	+0.06971	+0.24399	+0.43207	+0.51063
5.3.....	-0.27845	-0.13171	+0.03103	+0.20955	+0.40334	+0.48461
5.4.....	-0.32111	-0.17305	-0.00367	+0.17486	+0.37421	+0.45814
5.5.....	-0.36314	-0.21409	-0.04642	+0.13994	+0.34471	+0.43127
5.6.....	-0.40443	-0.25475	-0.08506	+0.10485	+0.31485	+0.40401
5.7.....	-0.44492	-0.29497	-0.12358	+0.06962	+0.28469	+0.37638
5.8.....	-0.48451	-0.33467	-0.16192	+0.03431	+0.25424	+0.34842
5.9.....	-0.52314	-0.37379	-0.20001	-0.00104	+0.22354	+0.32014
6.0.....	-0.56072	-0.41225	-0.23780	-0.03639	+0.19261	+0.29158
6.1.....	-0.59718	-0.44999	-0.27523	-0.07170	+0.16149	+0.26275
6.2.....	-0.63244	-0.48695	-0.31225	-0.10692	+0.13021	+0.23368
6.3.....	-0.66644	-0.52305	-0.34881	-0.14200	+0.09880	+0.20441
6.4.....	-0.69910	-0.55824	-0.38484	-0.17690	+0.06729	+0.17495
6.5.....	-0.73037	-0.59245	-0.42029	-0.21159	+0.03571	+0.14534
6.6.....	-0.76018	-0.62562	-0.45511	-0.11889	+0.00409	+0.11559
6.7.....	-0.78846	-0.65770	-0.48925	-0.28011	-0.02753	+0.08574
6.8.....	-0.81517	-0.68863	-0.52265	-0.31387	-0.05912	+0.05581
6.9.....	-0.84026	-0.71835	-0.55527	-0.34724	-0.09065	+0.02583
7.0.....	-0.86365	-0.74682	-0.58706	-0.38017	-0.12210	-0.00418
7.1.....	-0.88533	-0.77398	-0.61796	-0.41263	-0.15342	-0.03418
7.2.....	-0.90523	-0.79978	-0.64794	-0.44457	-0.18458	-0.06414
7.3.....	-0.92332	-0.82419	-0.67695	-0.47596	-0.21557	-0.09406
7.4.....	-0.93957	-0.84715	-0.70494	-0.50675	-0.24633	-0.12388
7.5.....	-0.95393	-0.86863	-0.73187	-0.53690	-0.27685	-0.15360
7.6.....	-0.96639	-0.88860	-0.75771	-0.56639	-0.30710	-0.18317
7.7.....	-0.97692	-0.90700	-0.78241	-0.59516	-0.33704	-0.21258
7.8.....	-0.98549	-0.92382	-0.80594	-0.62320	-0.36663	-0.24180
7.9.....	-0.99210	-0.93902	-0.82826	-0.65045	-0.39587	-0.27080
8.0.....	-0.99671	-0.95258	-0.84933	-0.67689	-0.42470	-0.29956
8.1.....	-0.99934	-0.96448	-0.86914	-0.70249	-0.45312	-0.32804
8.2.....	-0.99997	-0.97468	-0.88764	-0.72721	-0.48107	-0.35623
8.3.....	-0.99859	-0.98318	-0.90480	-0.75102	-0.50855	-0.38410
8.4.....	-0.99522	-0.98996	-0.92061	-0.77389	-0.53552	-0.41162
8.5.....	-0.98986	-0.99501	-0.93504	-0.79579	-0.56196	-0.43878
8.6.....	-0.98252	-0.99831	-0.94807	-0.81670	-0.58783	-0.46553
8.7.....	-0.97321	-0.99987	-0.95968	-0.83659	-0.61311	-0.49187
8.8.....	-0.96196	-0.99968	-0.96984	-0.85543	-0.63778	-0.51776
8.9.....	-0.94879	-0.99774	-0.97855	-0.87321	-0.66182	-0.54319
9.0.....	-0.93372	-0.99406	-0.98579	-0.88989	-0.68519	-0.56813
9.1.....	-0.91677	-0.98863	-0.99156	-0.90546	-0.70788	-0.59256
9.2.....	-0.89800	-0.98148	-0.99583	-0.91990	-0.72986	-0.61646
9.3.....	-0.87743	-0.97261	-0.99862	-0.93319	-0.75111	-0.63980
9.4.....	-0.85510	-0.96203	-0.99990	-0.94531	-0.77161	-0.66256
9.5.....	-0.83108	-0.94978	-0.99969	-0.95625	-0.79133	-0.68473
9.6.....	-0.80538	-0.93586	-0.99798	-0.96599	-0.81027	-0.70628
9.7.....	-0.77808	-0.92031	-0.99476	-0.97453	-0.82839	-0.72720
9.8.....	-0.74922	-0.90314	-0.99006	-0.98184	-0.84569	-0.74746
9.9.....	-0.71886	-0.88440	-0.98388	-0.98793	-0.86214	-0.76705
10.0.....	-0.68706	-0.86411	-0.97621	-0.99279	-0.87772	-0.78595

TABLE 5—Continued

$r$	$k^2 = 0.08$	$k^2 = 0.07$	$k^2 = 0.06$	$k^2 = 0.055$	$k^2 = 0.050$	$k^2 = 0.045$
0	0	0	0	0	0	0
0.1	+0.18134	+0.17931	+0.17699	+0.17561	+0.17396	+0.17215
0.2	+0.32977	+0.32610	+0.32190	+0.31940	+0.31641	+0.31312
0.3	+0.45132	+0.44634	+0.44064	+0.43723	+0.43316	+0.42867
0.4	+0.55097	+0.54497	+0.53808	+0.53395	+0.52902	+0.52358
0.5	+0.63279	+0.62601	+0.61821	+0.61352	+0.60792	+0.60172
0.6	+0.70008	+0.69274	+0.68427	+0.67917	+0.67304	+0.66625
0.7	+0.75552	+0.74782	+0.73889	+0.73348	+0.72697	+0.71975
0.8	+0.80127	+0.79338	+0.78418	+0.77857	+0.77180	+0.76426
0.9	+0.83907	+0.83115	+0.82185	+0.81614	+0.80920	+0.80146
1.0	+0.87032	+0.86251	+0.85326	+0.84753	+0.84053	+0.83269
1.1	+0.89614	+0.88859	+0.87953	+0.87386	+0.86687	+0.85901
1.2	+0.91744	+0.91027	+0.90153	+0.89599	+0.88910	+0.88130
1.3	+0.93495	+0.92827	+0.91999	+0.91464	+0.90791	+0.90025
1.4	+0.94923	+0.94317	+0.93546	+0.93037	+0.92388	+0.91642
1.5	+0.96077	+0.95544	+0.94842	+0.94364	+0.93744	+0.93026
1.6	+0.96995	+0.96545	+0.95923	+0.95482	+0.94898	+0.94213
1.7	+0.97705	+0.97350	+0.96818	+0.96421	+0.95878	+0.95232
1.8	+0.98234	+0.97983	+0.97553	+0.97204	+0.96709	+0.96107
1.9	+0.98600	+0.98464	+0.98145	+0.97851	+0.97408	+0.96858
2.0	+0.98820	+0.98809	+0.98612	+0.98377	+0.97991	+0.97498
2.1	+0.98906	+0.99030	+0.98965	+0.98795	+0.98472	+0.98040
2.2	+0.98869	+0.99137	+0.99215	+0.99114	+0.98860	+0.98495
2.3	+0.98717	+0.99140	+0.99369	+0.99343	+0.99163	+0.98870
2.4	+0.98458	+0.99044	+0.99436	+0.99489	+0.99387	+0.99172
2.5	+0.98097	+0.98856	+0.99419	+0.99557	+0.99539	+0.99406
2.6	+0.97639	+0.98580	+0.99324	+0.99551	+0.99622	+0.99577
2.7	+0.97088	+0.98220	+0.99153	+0.99475	+0.99640	+0.99687
2.8	+0.96448	+0.97779	+0.98911	+0.99332	+0.99595	+0.99741
2.9	+0.95720	+0.97259	+0.98600	+0.99125	+0.99491	+0.99739
3.0	+0.94908	+0.96664	+0.98222	+0.98855	+0.99329	+0.99685
3.1	+0.94014	+0.95995	+0.97778	+0.98524	+0.99111	+0.99579
3.2	+0.93039	+0.95253	+0.97270	+0.98133	+0.98837	+0.99423
3.3	+0.91986	+0.94440	+0.96700	+0.97685	+0.98510	+0.99217
3.4	+0.90857	+0.93558	+0.96068	+0.97179	+0.98130	+0.98964
3.5	+0.89652	+0.92608	+0.95375	+0.96617	+0.97698	+0.98663
3.6	+0.88373	+0.91591	+0.94623	+0.95999	+0.97215	+0.98315
3.7	+0.87022	+0.90507	+0.93813	+0.95327	+0.96682	+0.97921
3.8	+0.85599	+0.89359	+0.92945	+0.94601	+0.96099	+0.97481
3.9	+0.84108	+0.88148	+0.92019	+0.93822	+0.95467	+0.96997
4.0	+0.82548	+0.86873	+0.91038	+0.92990	+0.94785	+0.96468
4.1	+0.80920	+0.85538	+0.90001	+0.92106	+0.94056	+0.95894
4.2	+0.79228	+0.84141	+0.88909	+0.91171	+0.93279	+0.95277
4.3	+0.77472	+0.82685	+0.87764	+0.90185	+0.92455	+0.94616
4.4	+0.75654	+0.81171	+0.86565	+0.89150	+0.91584	+0.93912
4.5	+0.73775	+0.79600	+0.85314	+0.88064	+0.90667	+0.93166
4.6	+0.71836	+0.77973	+0.84011	+0.86931	+0.89704	+0.92378
4.7	+0.69840	+0.76291	+0.82658	+0.85749	+0.88697	+0.91547
4.8	+0.67788	+0.74556	+0.81256	+0.84520	+0.87644	+0.90676
4.9	+0.65681	+0.72768	+0.79804	+0.83244	+0.86548	+0.89763
5.0	+0.63522	+0.70929	+0.78304	+0.81922	+0.85409	+0.88810

TABLE 5—Continued

$r$	$k^2 = 0.08$	$k^2 = 0.07$	$k^2 = 0.06$	$k^2 = 0.055$	$k^2 = 0.050$	$k^2 = 0.045$
5.1.....	+0.61312	+0.69040	+0.76758	+0.80556	+0.84226	+0.87817
5.2.....	+0.59053	+0.67103	+0.75165	+0.79144	+0.83002	+0.86784
5.3.....	+0.56747	+0.65119	+0.73527	+0.77690	+0.81735	+0.85712
5.4.....	+0.54395	+0.63090	+0.71845	+0.76192	+0.80428	+0.84601
5.5.....	+0.52000	+0.61016	+0.70120	+0.74652	+0.79081	+0.83452
5.6.....	+0.49563	+0.58899	+0.68352	+0.73072	+0.77694	+0.82266
5.7.....	+0.47086	+0.56741	+0.66544	+0.71451	+0.76268	+0.81043
5.8.....	+0.44572	+0.54543	+0.64695	+0.69790	+0.74804	+0.79783
5.9.....	+0.42022	+0.52307	+0.62808	+0.68091	+0.73302	+0.78487
6.0.....	+0.39439	+0.50035	+0.60883	+0.66355	+0.71764	+0.77156
6.1.....	+0.36824	+0.47727	+0.58922	+0.64583	+0.70190	+0.75790
6.2.....	+0.34179	+0.45386	+0.56925	+0.62775	+0.68580	+0.74390
6.3.....	+0.31507	+0.43014	+0.54894	+0.60932	+0.66937	+0.72956
6.4.....	+0.28810	+0.40611	+0.52830	+0.59056	+0.65260	+0.71490
6.5.....	+0.26090	+0.38179	+0.50734	+0.57148	+0.63550	+0.69991
6.6.....	+0.23349	+0.35721	+0.48608	+0.55208	+0.61808	+0.68461
6.7.....	+0.20590	+0.33238	+0.46453	+0.53237	+0.60036	+0.66901
6.8.....	+0.17814	+0.30732	+0.44269	+0.51238	+0.58234	+0.65310
6.9.....	+0.15024	+0.28204	+0.42060	+0.49210	+0.56402	+0.63689
7.0.....	+0.12222	+0.25656	+0.39825	+0.47155	+0.54542	+0.62040
7.1.....	+0.09410	+0.23090	+0.37566	+0.45074	+0.52655	+0.60363
7.2.....	+0.06590	+0.20508	+0.35285	+0.42968	+0.50742	+0.58659
7.3.....	+0.03766	+0.17912	+0.32982	+0.40839	+0.48803	+0.56929
7.4.....	+0.00938	+0.15303	+0.30660	+0.38687	+0.46840	+0.55172
7.5.....	-0.01890	+0.12683	+0.28320	+0.36514	+0.44854	+0.53391
7.6.....	-0.04717	+0.10055	+0.25963	+0.34321	+0.42845	+0.51586
7.7.....	-0.07541	+0.07419	+0.23590	+0.32109	+0.40814	+0.49758
7.8.....	-0.10358	+0.04779	+0.21202	+0.29879	+0.38764	+0.47907
7.9.....	-0.13167	+0.02134	+0.18802	+0.27633	+0.36693	+0.46034
8.0.....	-0.15965	-0.00511	+0.16391	+0.25372	+0.34605	+0.44141
8.1.....	-0.18750	-0.03157	+0.13970	+0.23097	+0.32499	+0.42228
8.2.....	-0.21521	-0.05800	+0.11541	+0.20809	+0.30377	+0.40296
8.3.....	-0.24274	-0.08439	+0.09104	+0.18509	+0.28240	+0.38346
8.4.....	-0.27008	-0.11072	+0.06662	+0.16200	+0.26088	+0.36378
8.5.....	-0.29720	-0.13697	+0.04216	+0.13881	+0.23923	+0.34394
8.6.....	-0.32409	-0.16313	+0.01768	+0.11555	+0.21747	+0.32395
8.7.....	-0.35071	-0.18917	-0.00682	+0.09222	+0.19560	+0.30380
8.8.....	-0.37706	-0.21508	-0.03131	+0.06885	+0.17362	+0.28353
8.9.....	-0.40310	-0.24083	-0.05578	+0.04543	+0.15156	+0.26312
9.0.....	-0.42882	-0.26642	-0.08022	+0.02199	+0.12943	+0.24260
9.1.....	-0.45420	-0.29183	-0.10461	-0.00146	+0.10723	+0.22196
9.2.....	-0.47921	-0.31703	-0.12894	-0.02491	+0.08497	+0.20123
9.3.....	-0.50384	-0.34201	-0.15319	-0.04834	+0.06268	+0.18040
9.4.....	-0.52807	-0.36675	-0.17735	-0.07175	+0.04035	+0.15950
9.5.....	-0.55187	-0.39123	-0.20140	-0.09512	+0.01800	+0.13852
9.6.....	-0.57513	-0.41544	-0.22533	-0.11844	-0.00436	+0.11748
9.7.....	-0.59814	-0.43936	-0.24912	-0.14169	-0.02671	+0.09638
9.8.....	-0.62056	-0.46297	-0.27277	-0.16487	-0.04905	+0.07524
9.9.....	-0.64249	-0.48626	-0.29625	-0.18795	-0.07137	+0.05407
10.0.....	-0.66390	-0.50920	-0.31955	-0.21093	-0.09365	+0.03287

TABLE 5—Continued

$r$	$k^2 = 0.04$	$k^2 = 0.035$	$k^2 = 0.030$	$k^2 = 0.025$	$k^2 = 0.020$	$k^2 = 0.015$
0	0	0	0	0	0	0
0.1	+0.17002	+0.16746	+0.16420	+0.16009	+0.15457	+0.14651
0.2	+0.30925	+0.30461	+0.29868	+0.29121	+0.28119	+0.26654
0.3	+0.42340	+0.41707	+0.40896	+0.39876	+0.38505	+0.36500
0.4	+0.51717	+0.50947	+0.49960	+0.48717	+0.47045	+0.44599
0.5	+0.59440	+0.58562	+0.57432	+0.56008	+0.54091	+0.51284
0.6	+0.65823	+0.64858	+0.63614	+0.62044	+0.59928	+0.56824
0.7	+0.71118	+0.70085	+0.68751	+0.67065	+0.64786	+0.61440
0.8	+0.75529	+0.74446	+0.73041	+0.71262	+0.68852	+0.65307
0.9	+0.79221	+0.78101	+0.76643	+0.74791	+0.72276	+0.68570
1.0	+0.82327	+0.81182	+0.79685	+0.77778	+0.75180	+0.71341
1.1	+0.84952	+0.83793	+0.82270	+0.80322	+0.77659	+0.73714
1.2	+0.87182	+0.86019	+0.84481	+0.82506	+0.79794	+0.75763
1.3	+0.89086	+0.87927	+0.86385	+0.84393	+0.81646	+0.77548
1.4	+0.90720	+0.89572	+0.88034	+0.86036	+0.83266	+0.79116
1.5	+0.92127	+0.90999	+0.89473	+0.87478	+0.84696	+0.80507
1.6	+0.93344	+0.92243	+0.90736	+0.88752	+0.85967	+0.81751
1.7	+0.94399	+0.93331	+0.91851	+0.89886	+0.87107	+0.82875
1.8	+0.95316	+0.94287	+0.92841	+0.90902	+0.88136	+0.83899
1.9	+0.96114	+0.95130	+0.93724	+0.91817	+0.89073	+0.84838
2.0	+0.96807	+0.95875	+0.94515	+0.92647	+0.89931	+0.85706
2.1	+0.97409	+0.96533	+0.95225	+0.93402	+0.90721	+0.86514
2.2	+0.97928	+0.97114	+0.95864	+0.94093	+0.91453	+0.87271
2.3	+0.98373	+0.97626	+0.96440	+0.94726	+0.92134	+0.87983
2.4	+0.98749	+0.98077	+0.96959	+0.95309	+0.92770	+0.88656
2.5	+0.99064	+0.98469	+0.97426	+0.95845	+0.93365	+0.89296
2.6	+0.99319	+0.98809	+0.97846	+0.96339	+0.93924	+0.89905
2.7	+0.99520	+0.99099	+0.98222	+0.96795	+0.94450	+0.90487
2.8	+0.99669	+0.99342	+0.98555	+0.97214	+0.94945	+0.91044
2.9	+0.99768	+0.99540	+0.98849	+0.97599	+0.95412	+0.91578
3.0	+0.99819	+0.99695	+0.99106	+0.97951	+0.95851	+0.92091
3.1	+0.99823	+0.99809	+0.99326	+0.98273	+0.96265	+0.92584
3.2	+0.99782	+0.99882	+0.99511	+0.98565	+0.96655	+0.93058
3.3	+0.99697	+0.99916	+0.99662	+0.98828	+0.97021	+0.93514
3.4	+0.99568	+0.99912	+0.99780	+0.99063	+0.97365	+0.93952
3.5	+0.99396	+0.99869	+0.99864	+0.99270	+0.97686	+0.94374
3.6	+0.99183	+0.99794	+0.99917	+0.99450	+0.97985	+0.94780
3.7	+0.98927	+0.99683	+0.99937	+0.99603	+0.98262	+0.95170
3.8	+0.98631	+0.99535	+0.99926	+0.99730	+0.98519	+0.95543
3.9	+0.98294	+0.99351	+0.99884	+0.99830	+0.98754	+0.95902
4.0	+0.97916	+0.99131	+0.99811	+0.99905	+0.98969	+0.96244
4.1	+0.97499	+0.98875	+0.99707	+0.99954	+0.99163	+0.96572
4.2	+0.97042	+0.98585	+0.99572	+0.99977	+0.99337	+0.96885
4.3	+0.96545	+0.98259	+0.99407	+0.99974	+0.99490	+0.97182
4.4	+0.96010	+0.97898	+0.99212	+0.99946	+0.99623	+0.97464
4.5	+0.95435	+0.97503	+0.98986	+0.99893	+0.99735	+0.97732
4.6	+0.94822	+0.97074	+0.98731	+0.99815	+0.99828	+0.97984
4.7	+0.94171	+0.96610	+0.98446	+0.99711	+0.99900	+0.98222
4.8	+0.93483	+0.96112	+0.98131	+0.99582	+0.99951	+0.98444
4.9	+0.92756	+0.95580	+0.97786	+0.99428	+0.99983	+0.98652
5.0	+0.91993	+0.95015	+0.97412	+0.99249	+0.99995	+0.98845

TABLE 5—Continued

$r$	$k^2 = 0.04$	$k^2 = 0.035$	$k^2 = 0.030$	$k^2 = 0.025$	$k^2 = 0.020$	$k^2 = 0.015$
5.1.....	+0.91192	+0.94417	+0.97008	+0.99045	+0.99986	+0.99023
5.2.....	+0.90355	+0.93785	+0.96576	+0.98816	+0.99958	+0.99186
5.3.....	+0.89482	+0.93120	+0.96114	+0.98562	+0.99909	+0.99334
5.4.....	+0.88573	+0.92423	+0.95624	+0.98284	+0.99840	+0.99467
5.5.....	+0.87629	+0.91693	+0.95104	+0.97981	+0.99751	+0.99585
5.6.....	+0.86649	+0.90931	+0.94556	+0.97653	+0.99642	+0.99688
5.7.....	+0.85635	+0.90138	+0.93980	+0.97301	+0.99513	+0.99776
5.8.....	+0.84586	+0.89313	+0.93375	+0.96925	+0.99365	+0.99849
5.9.....	+0.83503	+0.88456	+0.92742	+0.96525	+0.99196	+0.99907
6.0.....	+0.82388	+0.87569	+0.92082	+0.96100	+0.99008	+0.99949
6.1.....	+0.81239	+0.86651	+0.91393	+0.95652	+0.98799	+0.99977
6.2.....	+0.80057	+0.85702	+0.90677	+0.95179	+0.98571	+0.99990
6.3.....	+0.78844	+0.84724	+0.89934	+0.94683	+0.98324	+0.99988
6.4.....	+0.77599	+0.83716	+0.89164	+0.94163	+0.98056	+0.99970
6.5.....	+0.76322	+0.82678	+0.88368	+0.93619	+0.97769	+0.99938
6.6.....	+0.75016	+0.81612	+0.87545	+0.93052	+0.97462	+0.99891
6.7.....	+0.73679	+0.80517	+0.86695	+0.92462	+0.97136	+0.99828
6.8.....	+0.72313	+0.79393	+0.85819	+0.91849	+0.96791	+0.99751
6.9.....	+0.70918	+0.78242	+0.84918	+0.91212	+0.96426	+0.99658
7.0.....	+0.69494	+0.77064	+0.83991	+0.90553	+0.96042	+0.99551
7.1.....	+0.68043	+0.75858	+0.83039	+0.89871	+0.95638	+0.99429
7.2.....	+0.66565	+0.74626	+0.82062	+0.89167	+0.95216	+0.99291
7.3.....	+0.65060	+0.73368	+0.81061	+0.88441	+0.94774	+0.99139
7.4.....	+0.63528	+0.72084	+0.80035	+0.87692	+0.94313	+0.98972
7.5.....	+0.61972	+0.70775	+0.78985	+0.86922	+0.93834	+0.98790
7.6.....	+0.60390	+0.69441	+0.77911	+0.86129	+0.93335	+0.98594
7.7.....	+0.58785	+0.68083	+0.76814	+0.85316	+0.92818	+0.98382
7.8.....	+0.57156	+0.66701	+0.75694	+0.84481	+0.92283	+0.98156
7.9.....	+0.55504	+0.65296	+0.74551	+0.83624	+0.91729	+0.97915
8.0.....	+0.53830	+0.63867	+0.73386	+0.82747	+0.91156	+0.97659
8.1.....	+0.52134	+0.62417	+0.72198	+0.81849	+0.90565	+0.97389
8.2.....	+0.50418	+0.60944	+0.70989	+0.80931	+0.89956	+0.97104
8.3.....	+0.48681	+0.59451	+0.69759	+0.79992	+0.89329	+0.96804
8.4.....	+0.46925	+0.57936	+0.68507	+0.79034	+0.88685	+0.96490
8.5.....	+0.45150	+0.56401	+0.67235	+0.78056	+0.88022	+0.96161
8.6.....	+0.43357	+0.54847	+0.65943	+0.77058	+0.87342	+0.95818
8.7.....	+0.41547	+0.53273	+0.64632	+0.76040	+0.86644	+0.95461
8.8.....	+0.39719	+0.51680	+0.63300	+0.75004	+0.85930	+0.95089
8.9.....	+0.37876	+0.50069	+0.61950	+0.73949	+0.85197	+0.94703
9.0.....	+0.36018	+0.48441	+0.60581	+0.72876	+0.84448	+0.94303
9.1.....	+0.34145	+0.46796	+0.59194	+0.71784	+0.83682	+0.93888
9.2.....	+0.32259	+0.45134	+0.57790	+0.70674	+0.82899	+0.93460
9.3.....	+0.30360	+0.43457	+0.56368	+0.69547	+0.82100	+0.93017
9.4.....	+0.28448	+0.41764	+0.54929	+0.68402	+0.81284	+0.92561
9.5.....	+0.26525	+0.40057	+0.53473	+0.67240	+0.80452	+0.92090
9.6.....	+0.24592	+0.38336	+0.52001	+0.66061	+0.79604	+0.91606
9.7.....	+0.22648	+0.36601	+0.50514	+0.64866	+0.78741	+0.91108
9.8.....	+0.20696	+0.34853	+0.49012	+0.63655	+0.77861	+0.90596
9.9.....	+0.18735	+0.33094	+0.47495	+0.62427	+0.76966	+0.90071
10.0.....	+0.16767	+0.31322	+0.45964	+0.61184	+0.76056	+0.89532

TABLE 6

THE RADIAL P-WAVE FUNCTIONS  $\chi_1$  OF AN ELECTRON  
IN THE HARTREE FIELD OF A HYDROGEN ATOM

$r$	$k^2 = 1.75$	$k^2 = 1.50$	$k^2 = 1.25$	$k^2 = 1.00$	$k^2 = 0.80$	$k^2 = 0.70$
0. ....	0	0	0	0	0	0
0.1. ....	+0.00941	+0.00821	+0.00691	+0.00557	+0.00447	+0.00392
0.2. ....	+0.03581	+0.03127	+0.02631	+0.02122	+0.01706	+0.01495
0.3. ....	+0.07662	+0.06700	+0.05645	+0.04559	+0.03669	+0.03216
0.4. ....	+0.12948	+0.11343	+0.09575	+0.07748	+0.06244	+0.05477
0.5. ....	+0.19220	+0.16879	+0.14282	+0.11585	+0.09354	+0.08213
0.6. ....	+0.26270	+0.23140	+0.19639	+0.15978	+0.12931	+0.11368
0.7. ....	+0.33898	+0.29968	+0.25527	+0.20843	+0.16916	+0.14893
0.8. ....	+0.41910	+0.37211	+0.31831	+0.26100	+0.21254	+0.18743
0.9. ....	+0.50119	+0.44722	+0.38444	+0.31675	+0.25893	+0.22878
1.0. ....	+0.58437	+0.52353	+0.45257	+0.37495	+0.30784	+0.27258
1.1. ....	+0.66585	+0.59964	+0.52165	+0.43487	+0.35878	+0.31845
1.2. ....	+0.74388	+0.67412	+0.59062	+0.49578	+0.41126	+0.36600
1.3. ....	+0.81675	+0.74562	+0.65845	+0.55698	+0.46479	+0.41486
1.4. ....	+0.88285	+0.81280	+0.72411	+0.61773	+0.51888	+0.46463
1.5. ....	+0.94069	+0.87439	+0.78660	+0.67732	+0.57303	+0.51492
1.6. ....	+0.98887	+0.92920	+0.84495	+0.73503	+0.62674	+0.56533
1.7. ....	+1.02618	+0.97611	+0.89823	+0.79017	+0.67950	+0.61545
1.8. ....	+1.05157	+1.01410	+0.94554	+0.84203	+0.73081	+0.66486
1.9. ....	+1.06416	+1.04229	+0.98607	+0.88996	+0.78016	+0.71317
2.0. ....	+1.06332	+1.05993	+1.01908	+0.93332	+0.82707	+0.75994
2.1. ....	+1.04863	+1.06639	+1.04389	+0.97150	+0.87103	+0.80478
2.2. ....	+1.01989	+1.06124	+1.05993	+1.00394	+0.91160	+0.84728
2.3. ....	+0.97718	+1.04418	+1.06673	+1.03014	+0.94832	+0.88704
2.4. ....	+0.92080	+1.01513	+1.06394	+1.04964	+0.98076	+0.92370
2.5. ....	+0.85131	+0.97416	+1.05130	+1.06205	+1.00854	+0.95688
2.6. ....	+0.76950	+0.92152	+1.02869	+1.06705	+1.03129	+0.98625
2.7. ....	+0.67641	+0.85765	+0.99613	+1.06437	+1.04868	+1.01149
2.8. ....	+0.57326	+0.78319	+0.95373	+1.05384	+1.06044	+1.03229
2.9. ....	+0.46150	+0.69891	+0.90176	+1.03537	+1.06632	+1.04841
3.0. ....	+0.34274	+0.60575	+0.84060	+1.00893	+1.06611	+1.05959
3.1. ....	+0.21873	+0.50484	+0.77075	+0.97458	+1.05969	+1.06565
3.2. ....	+0.09135	+0.39739	+0.69284	+0.93246	+1.04693	+1.06641
3.3. ....	-0.03746	+0.28474	+0.60760	+0.88281	+1.02781	+1.06174
3.4. ....	-0.16568	+0.16834	+0.51587	+0.82593	+1.00232	+1.05156
3.5. ....	-0.29128	+0.04970	+0.41858	+0.76220	+0.97052	+1.03581
3.6. ....	-0.41226	-0.06959	+0.31673	+0.69208	+0.93252	+1.01448
3.7. ....	-0.52666	-0.18796	+0.21141	+0.61609	+0.88849	+0.98760
3.8. ....	-0.63261	-0.30377	+0.10376	+0.53485	+0.83864	+0.95523
3.9. ....	-0.72838	-0.41545	-0.00505	+0.44899	+0.78323	+0.91749
4.0. ....	-0.81236	-0.52144	-0.11379	+0.35923	+0.72258	+0.87453
4.1. ....	-0.88315	-0.62027	-0.22126	+0.26633	+0.65706	+0.82654
4.2. ....	-0.93955	-0.71053	-0.32622	+0.17109	+0.58706	+0.77374
4.3. ....	-0.98059	-0.79095	-0.42748	+0.07432	+0.51302	+0.71640
4.4. ....	-1.00555	-0.86037	-0.52386	-0.02310	+0.43544	+0.65482
4.5. ....	-1.01397	-0.91779	-0.61423	-0.12032	+0.35483	+0.58934
4.6. ....	-1.00566	-0.96235	-0.69754	-0.21645	+0.27172	+0.52031
4.7. ....	-0.98073	-0.99340	-0.77279	-0.31062	+0.18671	+0.44813
4.8. ....	-0.93954	-1.01047	-0.83909	-0.40197	+0.10036	+0.37322
4.9. ....	-0.88275	-1.01327	-0.89563	-0.48965	+0.01331	+0.29603
5.0. ....	-0.81127	-1.00173	-0.94174	-0.57284	-0.07384	+0.21701

TABLE 6—Continued

$r$	$k^2 = 1.75$	$k^2 = 1.50$	$k^2 = 1.25$	$k^2 = 1.00$	$k^2 = 0.80$	$k^2 = 0.70$
5.1.....	-0.72625	-0.97598	-0.97684	-0.65077	-0.16046	+0.13664
5.2.....	-0.62910	-0.93636	-1.00049	-0.72269	-0.24592	+0.05543
5.3.....	-0.52143	-0.88340	-1.01238	-0.78792	-0.32960	-0.02613
5.4.....	-0.40501	-0.81784	-1.01235	-0.84584	-0.41087	-0.10753
5.5.....	-0.28179	-0.74058	-1.00037	-0.89589	-0.48914	-0.18825
5.6.....	-0.15383	-0.65272	-0.97656	-0.93758	-0.56382	-0.26778
5.7.....	-0.02329	-0.55549	-0.94118	-0.97050	-0.63435	-0.34560
5.8.....	+0.10765	-0.45028	-0.89462	-0.99431	-0.70021	-0.42122
5.9.....	+0.23677	-0.33860	-0.83742	-1.00878	-0.76087	-0.49414
6.0.....	+0.36189	-0.22203	-0.77025	-1.01375	-0.81589	-0.56388
6.1.....	+0.48089	-0.10226	-0.69388	-1.00915	-0.86485	-0.63000
6.2.....	+0.59174	+0.01898	-0.60923	-0.99501	-0.90735	-0.69204
6.3.....	+0.69255	+0.13995	-0.51728	-0.97145	-0.94307	-0.74961
6.4.....	+0.78161	+0.25890	-0.41913	-0.93867	-0.97172	-0.80230
6.5.....	+0.85740	+0.37409	-0.31595	-0.89697	-0.99308	-0.84978
6.6.....	+0.91860	+0.48385	-0.20898	-0.84673	-1.00697	-0.89171
6.7.....	+0.96417	+0.58658	-0.09950	-0.78841	-1.01327	-0.92781
6.8.....	+0.99333	+0.68079	+0.01119	-0.72257	-1.01192	-0.95783
6.9.....	+1.00555	+0.76509	+0.12174	-0.64982	-1.00292	-0.98157
7.0.....	+1.00063	+0.83825	+0.23082	-0.57086	-0.98632	-0.99885
7.1.....	+0.97862	+0.89920	+0.33711	-0.48642	-0.96223	-1.00955
7.2.....	+0.93990	+0.94702	+0.43932	-0.39731	-0.93084	-1.01359
7.3.....	+0.88512	+0.98103	+0.53622	-0.30439	-0.89236	-1.01093
7.4.....	+0.81521	+1.00070	+0.62662	-0.20854	-0.84709	-1.00157
7.5.....	+0.73134	+1.00575	+0.70943	-0.11068	-0.79535	-0.98557
7.6.....	+0.63496	+0.99609	+0.78363	-0.01175	-0.73753	-0.96303
7.7.....	+0.52770	+0.97185	+0.84831	+0.08728	-0.67407	-0.93408
7.8.....	+0.41139	+0.93337	+0.90269	+0.18548	-0.60545	-0.89891
7.9.....	+0.28804	+0.88122	+0.94609	+0.28188	-0.53219	-0.85775
8.0.....	+0.15974	+0.81615	+0.97798	+0.37556	-0.45485	-0.81087
8.1.....	+0.02869	+0.73911	+0.99797	+0.46560	-0.37401	-0.75856
8.2.....	-0.10284	+0.65122	+1.00579	+0.55113	-0.29029	-0.70118
8.3.....	-0.23261	+0.55376	+1.00136	+0.63132	-0.20434	-0.63910
8.4.....	-0.35838	+0.44817	+0.98471	+0.70538	-0.11682	-0.57273
8.5.....	-0.47799	+0.33599	+0.95604	+0.77259	-0.02839	-0.50252
8.6.....	-0.58938	+0.21888	+0.91570	+0.83230	+0.06025	-0.42894
8.7.....	-0.69062	+0.09854	+0.86417	+0.88392	+0.14843	-0.35247
8.8.....	-0.77999	-0.02325	+0.80208	+0.92693	+0.23547	-0.27362
8.9.....	-0.85592	-0.14469	+0.73018	+0.96093	+0.32068	-0.19294
9.0.....	-0.91711	-0.26400	+0.64934	+0.98557	+0.40340	-0.11095
9.1.....	-0.96250	-0.37943	+0.56056	+1.00060	+0.48300	-0.02821
9.2.....	-0.99130	-0.48926	+0.46491	+1.00587	+0.55886	+0.05472
9.3.....	-1.00302	-0.59187	+0.36357	+1.00133	+0.63038	+0.13728
9.4.....	-0.99744	-0.68576	+0.25777	+0.98702	+0.69701	+0.21891
9.5.....	-0.97465	-0.76952	+0.14881	+0.96307	+0.75822	+0.29905
9.6.....	-0.93505	-0.84193	+0.03803	+0.92971	+0.81354	+0.37717
9.7.....	-0.87931	-0.90191	-0.07322	+0.88726	+0.86252	+0.45274
9.8.....	-0.80839	-0.94856	-0.18358	+0.83614	+0.90480	+0.52523
9.9.....	-0.72351	-0.98121	-0.29167	+0.77683	+0.94003	+0.59416
10.0.....	-0.62614	-0.99935	-0.39619	+0.70992	+0.96794	+0.65905

TABLE 6—Continued

$r$	$k^2 = 0.60$	$k^2 = 0.50$	$k^2 = 0.45$	$k^2 = 0.35$	$k^2 = 0.25$	$k^2 = 0.20$
0	0	0	0	0	0	0
0.1	+0.00336	+0.00279	+0.00251	+0.00195	+0.00138	+0.00110
0.2	+0.01282	+0.01067	+0.00960	+0.00744	+0.00528	+0.00421
0.3	+0.02759	+0.02298	+0.02067	+0.01603	+0.01139	+0.00908
0.4	+0.04702	+0.03920	+0.03527	+0.02737	+0.01946	+0.01552
0.5	+0.07058	+0.05890	+0.05302	+0.04118	+0.02931	+0.02338
0.6	+0.09781	+0.08172	+0.07360	+0.05724	+0.04079	+0.03256
0.7	+0.12832	+0.10736	+0.09676	+0.07536	+0.05378	+0.04296
0.8	+0.16176	+0.13556	+0.12228	+0.09539	+0.06818	+0.05451
0.9	+0.19782	+0.16609	+0.14996	+0.11721	+0.08393	+0.06716
1.0	+0.23620	+0.19874	+0.17962	+0.14069	+0.10095	+0.08086
1.1	+0.27660	+0.23329	+0.21109	+0.16572	+0.11920	+0.09559
1.2	+0.31875	+0.26954	+0.24421	+0.19222	+0.13861	+0.11130
1.3	+0.36235	+0.30729	+0.27881	+0.22008	+0.15914	+0.12796
1.4	+0.40711	+0.34633	+0.31472	+0.24898	+0.18074	+0.14555
1.5	+0.45274	+0.38645	+0.35177	+0.27945	+0.20335	+0.16402
1.6	+0.49892	+0.42743	+0.38978	+0.31075	+0.22692	+0.18335
1.7	+0.54534	+0.46905	+0.42857	+0.34298	+0.25140	+0.20351
1.8	+0.59169	+0.51107	+0.46794	+0.37602	+0.27673	+0.22445
1.9	+0.63763	+0.55326	+0.50769	+0.40975	+0.30283	+0.24614
2.0	+0.68285	+0.59537	+0.54762	+0.44403	+0.32965	+0.26852
2.1	+0.72701	+0.63714	+0.58752	+0.47873	+0.35711	+0.29157
2.2	+0.76978	+0.67835	+0.62719	+0.51372	+0.38514	+0.31523
2.3	+0.81084	+0.71872	+0.66640	+0.54885	+0.41366	+0.33945
2.4	+0.84987	+0.75800	+0.70494	+0.58397	+0.44259	+0.36417
2.5	+0.88655	+0.79595	+0.74259	+0.61894	+0.47184	+0.38934
2.6	+0.92059	+0.83232	+0.77913	+0.65361	+0.50134	+0.41491
2.7	+0.95168	+0.86687	+0.81436	+0.68782	+0.53099	+0.44081
2.8	+0.97956	+0.89935	+0.84805	+0.72143	+0.56071	+0.46698
2.9	+1.00396	+0.92954	+0.88001	+0.75428	+0.59039	+0.49337
3.0	+1.02465	+0.95721	+0.91003	+0.78622	+0.61996	+0.51990
3.1	+1.04140	+0.98217	+0.93791	+0.81711	+0.64931	+0.54651
3.2	+1.05401	+1.00421	+0.96348	+0.84680	+0.67836	+0.57314
3.3	+1.06232	+1.02314	+0.98655	+0.87513	+0.70700	+0.59972
3.4	+1.06617	+1.03880	+1.00696	+0.90198	+0.73516	+0.62617
3.5	+1.06543	+1.05104	+1.02456	+0.92721	+0.76272	+0.65244
3.6	+1.06002	+1.05972	+1.03919	+0.95069	+0.78961	+0.67846
3.7	+1.04987	+1.06471	+1.05073	+0.97229	+0.81572	+0.70415
3.8	+1.13494	+1.06592	+1.05906	+0.99189	+0.84098	+0.72945
3.9	+1.01522	+1.06327	+1.06409	+1.00938	+0.86528	+0.75429
4.0	+0.99074	+1.05668	+1.06571	+1.02466	+0.88855	+0.77861
4.1	+0.96155	+1.04613	+1.06386	+1.03762	+0.91071	+0.80234
4.2	+0.92773	+1.03159	+1.05849	+1.04818	+0.93166	+0.82541
4.3	+0.88939	+1.01305	+1.04954	+1.05625	+0.95134	+0.84776
4.4	+0.84667	+0.99054	+1.03701	+1.06177	+0.96967	+0.86933
4.5	+0.79975	+0.96410	+1.02088	+1.06466	+0.98656	+0.89006
4.6	+0.74882	+0.93380	+1.00116	+1.06488	+1.00197	+0.90988
4.7	+0.69410	+0.89970	+0.97788	+1.06237	+1.01581	+0.92875
4.8	+0.63585	+0.86192	+0.95108	+1.05711	+1.02803	+0.94659
4.9	+0.57433	+0.82058	+0.92084	+1.04906	+1.03858	+0.96336
5.0	+0.50985	+0.77582	+0.88721	+1.03822	+1.04739	+0.97901

TABLE 6—Continued

$r$	$k^2 = 0.60$	$k^2 = 0.50$	$k^2 = 0.45$	$k^2 = 0.35$	$k^2 = 0.25$	$k^2 = 0.20$
5.1.....	+0.44272	+0.72780	+0.85030	+1.02457	+1.05442	+0.99348
5.2.....	+0.37328	+0.67670	+0.81022	+1.00812	+1.05962	+1.00672
5.3.....	+0.30187	+0.62272	+0.76710	+0.98889	+1.06296	+1.01870
5.4.....	+0.22887	+0.56608	+0.72106	+0.96690	+1.06439	+1.02936
5.5.....	+0.15465	+0.50699	+0.67228	+0.94219	+1.06390	+1.03867
5.6.....	+0.07960	+0.44570	+0.62092	+0.91481	+1.06145	+1.04659
5.7.....	+0.00413	+0.38247	+0.56717	+0.88481	+1.05702	+1.05309
5.8.....	-0.07136	+0.31756	+0.51121	+0.85226	+1.05060	+1.05812
5.9.....	-0.14647	+0.25125	+0.45325	+0.81723	+1.04218	+1.06167
6.0.....	-0.22078	+0.18383	+0.39351	+0.77982	+1.03176	+1.06371
6.1.....	-0.29389	+0.11560	+0.33223	+0.74010	+1.01932	+1.06421
6.2.....	-0.36540	+0.04685	+0.26963	+0.69820	+1.00489	+1.06315
6.3.....	-0.43490	-0.02211	+0.20595	+0.65421	+0.98847	+1.06052
6.4.....	-0.50202	-0.09097	+0.14145	+0.60827	+0.97007	+1.05631
6.5.....	-0.56636	-0.15942	+0.07639	+0.56049	+0.94973	+1.05049
6.6.....	-0.62758	-0.22714	+0.01102	+0.51102	+0.92746	+1.04308
6.7.....	-0.68533	-0.29384	-0.05440	+0.46000	+0.90329	+1.03405
6.8.....	-0.73927	-0.35920	-0.11959	+0.40757	+0.87728	+1.02342
6.9.....	-0.78909	-0.42292	-0.18430	+0.35389	+0.84944	+1.01119
7.0.....	-0.83452	-0.48470	-0.24826	+0.29912	+0.81985	+0.99736
7.1.....	-0.87528	-0.54426	-0.31120	+0.24343	+0.78853	+0.98194
7.2.....	-0.91113	-0.60131	-0.37286	+0.18698	+0.75556	+0.96495
7.3.....	-0.94188	-0.65559	-0.43299	+0.12995	+0.72099	+0.94639
7.4.....	-0.96733	-0.70684	-0.49134	+0.07251	+0.68489	+0.92631
7.5.....	-0.98733	-0.75481	-0.54765	+0.01485	+0.64733	+0.90471
7.6.....	-1.00176	-0.79928	-0.60170	-0.04286	+0.60838	+0.88162
7.7.....	-1.01053	-0.84003	-0.65325	-0.10044	+0.56813	+0.85707
7.8.....	-1.01358	-0.87687	-0.70208	-0.15769	+0.52664	+0.83110
7.9.....	-1.01088	-0.90961	-0.74798	-0.21445	+0.48401	+0.80374
8.0.....	-1.00245	-0.93809	-0.79075	-0.27053	+0.44032	+0.77503
8.1.....	-0.98832	-0.96218	-0.83022	-0.32574	+0.39567	+0.74501
8.2.....	-0.96856	-0.98176	-0.86620	-0.37991	+0.35016	+0.71373
8.3.....	-0.94328	-0.99672	-0.89855	-0.43287	+0.30387	+0.68123
8.4.....	-0.91262	-1.00698	-0.92711	-0.48444	+0.25691	+0.64757
8.5.....	-0.87674	-1.01250	-0.95177	-0.53444	+0.20938	+0.61280
8.6.....	-0.83585	-1.01324	-0.97241	-0.58273	+0.16139	+0.57698
8.7.....	-0.79017	-1.00919	-0.98893	-0.62914	+0.11303	+0.54015
8.8.....	-0.73996	-1.00036	-1.00127	-0.67351	+0.06443	+0.50239
8.9.....	-0.68550	-0.98679	-1.00937	-0.71570	+0.01568	+0.46375
9.0.....	-0.62711	-0.96854	-1.01317	-0.75556	-0.03311	+0.42431
9.1.....	-0.56511	-0.94568	-1.01267	-0.79297	-0.08182	+0.38411
9.2.....	-0.49986	-0.91833	-1.00786	-0.82779	-0.13035	+0.34325
9.3.....	-0.43173	-0.88661	-0.99876	-0.85991	-0.17858	+0.30178
9.4.....	-0.36110	-0.85065	-0.98539	-0.88922	-0.22640	+0.25977
9.5.....	-0.28840	-0.81064	-0.96781	-0.91563	-0.27371	+0.21731
9.6.....	-0.21403	-0.76676	-0.94609	-0.93903	-0.32040	+0.17445
9.7.....	-0.13842	-0.71921	-0.92033	-0.95935	-0.36636	+0.13129
9.8.....	-0.06201	-0.66822	-0.89061	-0.97651	-0.41148	+0.08789
9.9.....	+0.01475	-0.61403	-0.85708	-0.99046	-0.45565	+0.04434
10.0.....	+0.09143	-0.55689	-0.81987	-1.00115	-0.49878	+0.00070

TABLE 6—Continued

$r$	$k^2 = 0.175$	$k^2 = 0.150$	$k^2 = 0.125$	$k^2 = 0.10$	$k^2 = 0.09$	$k^2 = 0.08$
0	0	0	0	0	0	0
0.1	+0.00096	+0.00082	+0.00068	+0.00054	+0.00049	+0.00043
0.2	+0.00367	+0.00314	+0.00261	+0.00208	+0.00187	+0.00166
0.3	+0.00793	+0.00678	+0.00563	+0.00449	+0.00404	+0.00358
0.4	+0.01355	+0.01159	+0.00963	+0.00769	+0.00691	+0.00613
0.5	+0.02042	+0.01747	+0.01453	+0.01159	+0.01042	+0.00925
0.6	+0.02845	+0.02434	+0.02024	+0.01616	+0.01453	+0.01290
0.7	+0.03754	+0.03213	+0.02674	+0.02135	+0.01920	+0.01705
0.8	+0.04766	+0.04081	+0.03397	+0.02713	+0.02440	+0.02168
0.9	+0.05875	+0.05033	+0.04191	+0.03349	+0.03013	+0.02677
1.0	+0.07077	+0.06066	+0.05054	+0.04041	+0.03636	+0.03231
1.1	+0.08371	+0.07179	+0.05984	+0.04788	+0.04309	+0.03830
1.2	+0.09753	+0.08369	+0.06981	+0.05589	+0.05031	+0.04473
1.3	+0.11220	+0.09635	+0.08043	+0.06444	+0.05802	+0.05159
1.4	+0.12772	+0.10976	+0.09169	+0.07351	+0.06621	+0.05889
1.5	+0.14405	+0.12389	+0.10357	+0.08310	+0.07487	+0.06662
1.6	+0.16116	+0.13873	+0.11608	+0.09322	+0.08401	+0.07478
1.7	+0.17905	+0.15426	+0.12919	+0.10384	+0.09362	+0.08337
1.8	+0.19766	+0.17047	+0.14290	+0.11497	+0.10370	+0.09237
1.9	+0.21698	+0.18733	+0.15720	+0.12660	+0.11423	+0.10180
2.0	+0.23698	+0.20481	+0.17206	+0.13872	+0.12522	+0.11163
2.1	+0.25762	+0.22290	+0.18746	+0.15131	+0.13665	+0.12188
2.2	+0.27886	+0.24157	+0.20341	+0.16437	+0.14851	+0.13252
2.3	+0.30066	+0.26079	+0.21986	+0.17789	+0.16080	+0.14356
2.4	+0.32299	+0.28052	+0.23681	+0.19185	+0.17351	+0.15499
2.5	+0.34579	+0.30075	+0.25423	+0.20624	+0.18663	+0.16679
2.6	+0.36903	+0.32142	+0.27210	+0.22105	+0.20014	+0.17897
2.7	+0.39267	+0.34252	+0.29039	+0.23625	+0.21403	+0.19150
2.8	+0.41664	+0.36400	+0.30907	+0.25184	+0.22829	+0.20438
2.9	+0.44090	+0.38583	+0.32813	+0.26779	+0.24290	+0.21760
3.0	+0.46541	+0.40797	+0.34754	+0.28409	+0.25785	+0.23114
3.1	+0.49011	+0.43037	+0.36725	+0.30072	+0.27313	+0.24500
3.2	+0.51494	+0.45300	+0.38725	+0.31765	+0.28871	+0.25915
3.3	+0.53985	+0.47581	+0.40751	+0.33488	+0.30459	+0.27360
3.4	+0.56479	+0.49877	+0.42800	+0.35237	+0.32074	+0.28832
3.5	+0.58970	+0.52183	+0.44867	+0.37011	+0.33714	+0.30330
3.6	+0.61453	+0.54494	+0.46951	+0.38808	+0.35379	+0.31852
3.7	+0.63923	+0.56807	+0.49047	+0.40625	+0.37065	+0.33398
3.8	+0.66372	+0.59117	+0.51153	+0.42460	+0.38772	+0.34965
3.9	+0.68797	+0.61419	+0.53266	+0.44311	+0.40497	+0.36552
4.0	+0.71191	+0.63709	+0.55381	+0.46175	+0.42239	+0.38157
4.1	+0.73549	+0.65983	+0.57496	+0.48051	+0.43995	+0.39779
4.2	+0.75865	+0.68236	+0.59607	+0.49935	+0.45763	+0.41417
4.3	+0.78134	+0.70463	+0.61711	+0.51826	+0.47542	+0.43068
4.4	+0.80350	+0.72661	+0.63804	+0.53720	+0.49329	+0.44731
4.5	+0.82508	+0.74825	+0.65883	+0.55617	+0.51123	+0.46405
4.6	+0.84604	+0.76949	+0.67944	+0.57512	+0.52921	+0.48086
4.7	+0.86631	+0.79031	+0.69985	+0.59404	+0.54721	+0.49775
4.8	+0.88584	+0.81066	+0.72001	+0.61290	+0.56522	+0.51469
4.9	+0.90460	+0.83050	+0.73990	+0.63169	+0.58320	+0.53166
5.0	+0.92252	+0.84978	+0.75947	+0.65036	+0.60115	+0.54865

TABLE 6—Continued

$r$	$k^2 = 0.175$	$k^2 = 0.150$	$k^2 = 0.125$	$k^2 = 0.10$	$k^2 = 0.09$	$k^2 = 0.08$
5.1.....	+0.93957	+0.86846	+0.77871	+0.66891	+0.61903	+0.56564
5.2.....	+0.95569	+0.88651	+0.79757	+0.68730	+0.63684	+0.58261
5.3.....	+0.97085	+0.90389	+0.81602	+0.70551	+0.65454	+0.59955
5.4.....	+0.98500	+0.92055	+0.83403	+0.72351	+0.67211	+0.61643
5.5.....	+0.99810	+0.93646	+0.85158	+0.74129	+0.68955	+0.63325
5.6.....	+1.01011	+0.95158	+0.86862	+0.75882	+0.70681	+0.64997
5.7.....	+1.02100	+0.96589	+0.88512	+0.77607	+0.72389	+0.66659
5.8.....	+1.03073	+0.97934	+0.90107	+0.79303	+0.74077	+0.68308
5.9.....	+1.03927	+0.99190	+0.91643	+0.80966	+0.75742	+0.69944
6.0.....	+1.04659	+1.00355	+0.93116	+0.82595	+0.77382	+0.71563
6.1.....	+1.05266	+1.01425	+0.94525	+0.84187	+0.78996	+0.73166
6.2.....	+1.05746	+1.02397	+0.95867	+0.85740	+0.80581	+0.74748
6.3.....	+1.06095	+1.03269	+0.97139	+0.87252	+0.82135	+0.76311
6.4.....	+1.06312	+1.04038	+0.98338	+0.88720	+0.83657	+0.77850
6.5.....	+1.06395	+1.04701	+0.99462	+0.90144	+0.85145	+0.79365
6.6.....	+1.06342	+1.05258	+1.00509	+0.91519	+0.86596	+0.80854
6.7.....	+1.06152	+1.05704	+1.01477	+0.92846	+0.88009	+0.82316
6.8.....	+1.05823	+1.06040	+1.02363	+0.94120	+0.89382	+0.83749
6.9.....	+1.05355	+1.06262	+1.03165	+0.95342	+0.90713	+0.85150
7.0.....	+1.04747	+1.06369	+1.03882	+0.96508	+0.92000	+0.86520
7.1.....	+1.03998	+1.06360	+1.04511	+0.97617	+0.93243	+0.87855
7.2.....	+1.03109	+1.06234	+1.05051	+0.98667	+0.94438	+0.89155
7.3.....	+1.02079	+1.05990	+1.05500	+0.99657	+0.95585	+0.90418
7.4.....	+1.00909	+1.05626	+1.05857	+1.00584	+0.96682	+0.91643
7.5.....	+0.99599	+1.05143	+1.06121	+1.01447	+0.97727	+0.92828
7.6.....	+0.98150	+1.04539	+1.06289	+1.02245	+0.98718	+0.93972
7.7.....	+0.96563	+1.03814	+1.06361	+1.02977	+0.99655	+0.95073
7.8.....	+0.94840	+1.02969	+1.06337	+1.03640	+1.00537	+0.96130
7.9.....	+0.92983	+1.02003	+1.06214	+1.04233	+1.01360	+0.97141
8.0.....	+0.90992	+1.00917	+1.05993	+1.04756	+1.02125	+0.98106
8.1.....	+0.88870	+0.99712	+1.05672	+1.05206	+1.02830	+0.99024
8.2.....	+0.86621	+0.98387	+1.05252	+1.05584	+1.03474	+0.99892
8.3.....	+0.84245	+0.96943	+1.04731	+1.05887	+1.04055	+1.00710
8.4.....	+0.81746	+0.95383	+1.04109	+1.06115	+1.04573	+1.01477
8.5.....	+0.79128	+0.93706	+1.03387	+1.06267	+1.05026	+1.02191
8.6.....	+0.76393	+0.91915	+1.02565	+1.06342	+1.05414	+1.02852
8.7.....	+0.73545	+0.90011	+1.01642	+1.06340	+1.05736	+1.03458
8.8.....	+0.70587	+0.87995	+1.00619	+1.06259	+1.05990	+1.04009
8.9.....	+0.67525	+0.85871	+0.99496	+1.06100	+1.06177	+1.04504
9.0.....	+0.64361	+0.83639	+0.98274	+1.05861	+1.06295	+1.04941
9.1.....	+0.61101	+0.81302	+0.96953	+1.05543	+1.06343	+1.05321
9.2.....	+0.57748	+0.78864	+0.95534	+1.05145	+1.06321	+1.05641
9.3.....	+0.54308	+0.76325	+0.94019	+1.04666	+1.06229	+1.05902
9.4.....	+0.50786	+0.73690	+0.92408	+1.04107	+1.06065	+1.06103
9.5.....	+0.47186	+0.70961	+0.90702	+1.03467	+1.05831	+1.06243
9.6.....	+0.43514	+0.68141	+0.88903	+1.02747	+1.05524	+1.06321
9.7.....	+0.39775	+0.65233	+0.87012	+1.01946	+1.05145	+1.06338
9.8.....	+0.35975	+0.62242	+0.85031	+1.01065	+1.04694	+1.06292
9.9.....	+0.32120	+0.59170	+0.82962	+1.00104	+1.04171	+1.06183
10.0.....	+0.28215	+0.56022	+0.80805	+0.99063	+1.03575	+1.06011

TABLE 6—Continued

$r$	$k^2 = 0.07$	$k^2 = 0.06$	$k^2 = 0.055$	$k^2 = 0.050$	$k^2 = 0.045$	$k^2 = 0.040$
0. ....	0	0	0	0	0	0
0.1. ....	+0.00038	+0.00032	+0.00030	+0.00027	+0.00024	+0.00022
0.2. ....	+0.00145	+0.00124	+0.00114	+0.00103	+0.00093	+0.00083
0.3. ....	+0.00313	+0.00268	+0.00246	+0.00223	+0.00201	+0.00178
0.4. ....	+0.00536	+0.00459	+0.00420	+0.00382	+0.00343	+0.00305
0.5. ....	+0.00809	+0.00692	+0.00634	+0.00576	+0.00518	+0.00460
0.6. ....	+0.01128	+0.00965	+0.00884	+0.00804	+0.00723	+0.00642
0.7. ....	+0.01490	+0.01276	+0.01169	+0.01062	+0.00956	+0.00849
0.8. ....	+0.01895	+0.01623	+0.01487	+0.01351	+0.01216	+0.01080
0.9. ....	+0.02341	+0.02005	+0.01837	+0.01670	+0.01502	+0.01335
1.0. ....	+0.02826	+0.02421	+0.02219	+0.02017	+0.01815	+0.01613
1.1. ....	+0.03351	+0.02871	+0.02632	+0.02392	+0.02153	+0.01913
1.2. ....	+0.03914	+0.03355	+0.03075	+0.02796	+0.02516	+0.02237
1.3. ....	+0.04516	+0.03872	+0.03550	+0.03228	+0.02905	+0.02583
1.4. ....	+0.05157	+0.04423	+0.04055	+0.03688	+0.03320	+0.02952
1.5. ....	+0.05835	+0.05006	+0.04591	+0.04176	+0.03760	+0.03343
1.6. ....	+0.06552	+0.05623	+0.05158	+0.04691	+0.04225	+0.03758
1.7. ....	+0.07307	+0.06273	+0.05755	+0.05236	+0.04716	+0.04195
1.8. ....	+0.08099	+0.06956	+0.06383	+0.05808	+0.05233	+0.04655
1.9. ....	+0.08929	+0.07672	+0.07041	+0.06409	+0.05774	+0.05139
2.0. ....	+0.09796	+0.08421	+0.07730	+0.07037	+0.06342	+0.05645
2.1. ....	+0.10700	+0.09202	+0.08449	+0.07693	+0.06935	+0.06174
2.2. ....	+0.11640	+0.10015	+0.09197	+0.08376	+0.07553	+0.06726
2.3. ....	+0.12616	+0.10860	+0.09976	+0.09087	+0.08196	+0.07300
2.4. ....	+0.13627	+0.11736	+0.10783	+0.09826	+0.08864	+0.07897
2.5. ....	+0.14673	+0.12643	+0.11620	+0.10591	+0.09557	+0.08517
2.6. ....	+0.15752	+0.13581	+0.12485	+0.11383	+0.10274	+0.09158
2.7. ....	+0.16865	+0.14549	+0.13379	+0.12201	+0.11015	+0.09822
2.8. ....	+0.18010	+0.15546	+0.14300	+0.13044	+0.11781	+0.10507
2.9. ....	+0.19187	+0.16571	+0.15248	+0.13914	+0.12570	+0.11215
3.0. ....	+0.20394	+0.17625	+0.16223	+0.14808	+0.13382	+0.11943
3.1. ....	+0.21631	+0.18707	+0.17224	+0.15727	+0.14217	+0.12692
3.2. ....	+0.22896	+0.19815	+0.18250	+0.16670	+0.15074	+0.13462
3.3. ....	+0.24190	+0.20949	+0.19302	+0.17636	+0.15953	+0.14252
3.4. ....	+0.25510	+0.22108	+0.20377	+0.18625	+0.16854	+0.15063
3.5. ....	+0.26855	+0.23292	+0.21476	+0.19637	+0.17776	+0.15893
3.6. ....	+0.28225	+0.24499	+0.22597	+0.20670	+0.18719	+0.16742
3.7. ....	+0.29618	+0.25728	+0.23741	+0.21725	+0.19682	+0.17610
3.8. ....	+0.31034	+0.26980	+0.24906	+0.22800	+0.20664	+0.18496
3.9. ....	+0.32470	+0.28252	+0.26091	+0.23895	+0.21665	+0.19400
4.0. ....	+0.33926	+0.29544	+0.27296	+0.25010	+0.22685	+0.20322
4.1. ....	+0.35400	+0.30855	+0.28520	+0.26142	+0.23723	+0.21261
4.2. ....	+0.36891	+0.32184	+0.29762	+0.27293	+0.24778	+0.22216
4.3. ....	+0.38398	+0.33530	+0.31021	+0.28461	+0.25850	+0.23188
4.4. ....	+0.39919	+0.34892	+0.32297	+0.29645	+0.26939	+0.24176
4.5. ....	+0.41454	+0.36269	+0.33587	+0.30845	+0.28042	+0.25178
4.6. ....	+0.43001	+0.37660	+0.34893	+0.32060	+0.29161	+0.26196
4.7. ....	+0.44557	+0.39064	+0.36212	+0.33289	+0.30295	+0.27227
4.8. ....	+0.46123	+0.40480	+0.37544	+0.34532	+0.31441	+0.28273
4.9. ....	+0.47697	+0.41907	+0.38888	+0.35787	+0.32602	+0.29331
5.0. ....	+0.49277	+0.43343	+0.40243	+0.37054	+0.33774	+0.30402

TABLE 6—Continued

$r$	$k^2 = 0.07$	$k^2 = 0.06$	$k^2 = 0.055$	$k^2 = 0.050$	$k^2 = 0.045$	$k^2 = 0.040$
5.1.....	+0.50862	+0.44788	+0.41608	+0.38332	+0.34958	+0.31486
5.2.....	+0.52450	+0.46240	+0.42982	+0.39620	+0.36154	+0.32580
5.3.....	+0.54040	+0.47699	+0.44364	+0.40918	+0.37359	+0.33686
5.4.....	+0.55631	+0.49163	+0.45754	+0.42225	+0.38575	+0.34803
5.5.....	+0.57221	+0.50632	+0.47149	+0.43539	+0.39800	+0.35929
5.6.....	+0.58809	+0.52103	+0.48550	+0.44860	+0.41033	+0.37065
5.7.....	+0.60393	+0.53576	+0.49955	+0.46188	+0.42274	+0.38209
5.8.....	+0.61972	+0.55051	+0.51363	+0.47521	+0.43521	+0.39362
5.9.....	+0.63545	+0.56524	+0.52773	+0.48858	+0.44775	+0.40522
6.0.....	+0.65109	+0.57997	+0.54185	+0.50199	+0.46035	+0.41690
6.1.....	+0.66664	+0.59467	+0.55597	+0.51542	+0.47299	+0.42864
6.2.....	+0.68208	+0.60933	+0.57008	+0.52888	+0.48568	+0.44044
6.3.....	+0.69740	+0.62394	+0.58418	+0.54235	+0.49840	+0.45229
6.4.....	+0.71259	+0.63849	+0.59825	+0.55582	+0.51115	+0.46418
6.5.....	+0.72762	+0.65297	+0.61228	+0.56928	+0.52391	+0.47612
6.6.....	+0.74249	+0.66737	+0.62627	+0.58273	+0.53669	+0.48810
6.7.....	+0.75717	+0.68168	+0.64020	+0.59615	+0.54947	+0.50010
6.8.....	+0.77167	+0.69588	+0.65406	+0.60954	+0.56225	+0.51213
6.9.....	+0.78596	+0.70996	+0.66784	+0.62289	+0.57503	+0.52417
7.0.....	+0.80003	+0.72391	+0.68154	+0.63619	+0.58778	+0.53622
7.1.....	+0.81387	+0.73773	+0.69514	+0.64943	+0.60051	+0.54828
7.2.....	+0.82745	+0.75140	+0.70864	+0.66261	+0.61321	+0.56033
7.3.....	+0.84078	+0.76490	+0.72202	+0.67571	+0.62586	+0.57238
7.4.....	+0.85384	+0.77823	+0.73527	+0.68872	+0.63847	+0.58441
7.5.....	+0.86661	+0.79138	+0.74839	+0.70164	+0.65103	+0.59643
7.6.....	+0.87908	+0.80434	+0.76136	+0.71446	+0.66353	+0.60841
7.7.....	+0.89124	+0.81709	+0.77417	+0.72717	+0.67595	+0.62037
7.8.....	+0.90308	+0.82963	+0.78682	+0.73977	+0.68830	+0.63228
7.9.....	+0.91458	+0.84195	+0.79930	+0.75223	+0.70057	+0.64415
8.0.....	+0.92574	+0.85402	+0.81159	+0.76456	+0.71275	+0.65597
8.1.....	+0.93653	+0.86586	+0.82370	+0.77675	+0.72482	+0.66773
8.2.....	+0.94696	+0.87743	+0.83559	+0.78878	+0.73680	+0.67943
8.3.....	+0.95700	+0.88874	+0.84728	+0.80066	+0.74866	+0.69105
8.4.....	+0.96666	+0.89978	+0.85875	+0.81237	+0.76040	+0.70261
8.5.....	+0.97591	+0.91053	+0.86999	+0.82390	+0.77201	+0.71408
8.6.....	+0.98475	+0.92099	+0.88099	+0.83526	+0.78349	+0.72546
8.7.....	+0.99316	+0.93114	+0.89175	+0.84638	+0.79483	+0.73675
8.8.....	+1.00114	+0.94108	+0.90225	+0.85736	+0.80603	+0.74793
8.9.....	+1.00868	+0.95050	+0.91248	+0.86811	+0.81706	+0.75902
9.0.....	+1.01577	+0.95969	+0.92245	+0.87865	+0.82794	+0.76999
9.1.....	+1.02240	+0.96854	+0.93214	+0.88896	+0.83865	+0.78084
9.2.....	+1.02856	+0.97704	+0.94153	+0.89904	+0.84918	+0.79157
9.3.....	+1.03424	+0.98519	+0.95064	+0.90889	+0.85953	+0.80217
9.4.....	+1.03944	+0.99297	+0.95944	+0.91849	+0.86970	+0.81263
9.5.....	+1.04414	+1.00039	+0.96793	+0.92784	+0.87967	+0.82295
9.6.....	+1.04835	+1.00742	+0.97610	+0.93693	+0.88944	+0.83313
9.7.....	+1.05202	+1.01407	+0.98395	+0.94575	+0.89900	+0.84315
9.8.....	+1.05523	+1.02033	+0.99146	+0.95431	+0.90835	+0.85302
9.9.....	+1.05790	+1.02618	+0.99864	+0.96259	+0.91747	+0.86272
10.0.....	+1.06005	+1.03163	+1.00547	+0.97058	+0.92638	+0.87225

TABLE 6—Continued

$r$	$k^2 = 0.035$	$k^2 = 0.030$	$k^2 = 0.025$	$k^2 = 0.020$	$k^2 = 0.015$
0	0	0	0	0	0
0.1	+0.00019	+0.00016	+0.00013	+0.00011	+0.00008
0.2	+0.00072	+0.00062	+0.00051	+0.00041	+0.00031
0.3	+0.00156	+0.00133	+0.00111	+0.00089	+0.00067
0.4	+0.00267	+0.00228	+0.00190	+0.00152	+0.00114
0.5	+0.00402	+0.00345	+0.00287	+0.00229	+0.00172
0.6	+0.00561	+0.00481	+0.00401	+0.00320	+0.00240
0.7	+0.00742	+0.00636	+0.00530	+0.00424	+0.00317
0.8	+0.00945	+0.00809	+0.00674	+0.00539	+0.00404
0.9	+0.01167	+0.01000	+0.00833	+0.00666	+0.00499
1.0	+0.01411	+0.01209	+0.01007	+0.00805	+0.00604
1.1	+0.01674	+0.01434	+0.01195	+0.00956	+0.00717
1.2	+0.01957	+0.01677	+0.01398	+0.01118	+0.00838
1.3	+0.02260	+0.01937	+0.01615	+0.01292	+0.00969
1.4	+0.02583	+0.02215	+0.01846	+0.01477	+0.01108
1.5	+0.02927	+0.02509	+0.02092	+0.01674	+0.01256
1.6	+0.03290	+0.02822	+0.02352	+0.01883	+0.01413
1.7	+0.03674	+0.03151	+0.02628	+0.02104	+0.01579
1.8	+0.04077	+0.03498	+0.02918	+0.02336	+0.01754
1.9	+0.04501	+0.03863	+0.03222	+0.02581	+0.01938
2.0	+0.04946	+0.04245	+0.03542	+0.02837	+0.02131
2.1	+0.05411	+0.04645	+0.03877	+0.03106	+0.02333
2.2	+0.05895	+0.05062	+0.04226	+0.03387	+0.02544
2.3	+0.06401	+0.05497	+0.04590	+0.03679	+0.02765
2.4	+0.06926	+0.05950	+0.04969	+0.03984	+0.02995
2.5	+0.07471	+0.06420	+0.05363	+0.04301	+0.03234
2.6	+0.08036	+0.06907	+0.05772	+0.04631	+0.03482
2.7	+0.08621	+0.07412	+0.06196	+0.04972	+0.03740
2.8	+0.09225	+0.07934	+0.06634	+0.05325	+0.04007
2.9	+0.09849	+0.08473	+0.07087	+0.05690	+0.04283
3.0	+0.10492	+0.09029	+0.07554	+0.06067	+0.04568
3.1	+0.11154	+0.09602	+0.08036	+0.06456	+0.04863
3.2	+0.11835	+0.10191	+0.08532	+0.06857	+0.05166
3.3	+0.12534	+0.10797	+0.09042	+0.07270	+0.05479
3.4	+0.13251	+0.11419	+0.09566	+0.07694	+0.05801
3.5	+0.13986	+0.12057	+0.10105	+0.08130	+0.06132
3.6	+0.14739	+0.12710	+0.10656	+0.08577	+0.06471
3.7	+0.15509	+0.13380	+0.11222	+0.09035	+0.06820
3.8	+0.16296	+0.14064	+0.11801	+0.09505	+0.07177
3.9	+0.17100	+0.14764	+0.12393	+0.09986	+0.07543
4.0	+0.17920	+0.15478	+0.12998	+0.10478	+0.07918
4.1	+0.18756	+0.16208	+0.13616	+0.10981	+0.08302
4.2	+0.19607	+0.16951	+0.14247	+0.11494	+0.08694
4.3	+0.20474	+0.17708	+0.14890	+0.12019	+0.09095
4.4	+0.21356	+0.18480	+0.15545	+0.12554	+0.09504
4.5	+0.22252	+0.19264	+0.16213	+0.13099	+0.09921
4.6	+0.23163	+0.20062	+0.16893	+0.13654	+0.10347
4.7	+0.24087	+0.20873	+0.17584	+0.14220	+0.10781
4.8	+0.25024	+0.21696	+0.18287	+0.14796	+0.11223
4.9	+0.25975	+0.22532	+0.19001	+0.15381	+0.11673
5.0	+0.26938	+0.23379	+0.19726	+0.15977	+0.12131

TABLE 6—Continued

$r$	$k^2 = 0.035$	$k^2 = 0.030$	$k^2 = 0.025$	$k^2 = 0.020$	$k^2 = 0.015$
5.1.....	+0.27913	+0.24239	+0.20462	+0.16582	+0.12596
5.2.....	+0.28899	+0.25109	+0.21208	+0.17196	+0.13070
5.3.....	+0.29897	+0.25991	+0.21965	+0.17819	+0.13551
5.4.....	+0.30906	+0.26883	+0.22732	+0.18452	+0.14040
5.5.....	+0.31925	+0.27786	+0.23509	+0.19094	+0.14537
5.6.....	+0.32954	+0.28699	+0.24296	+0.19744	+0.15041
5.7.....	+0.33993	+0.29621	+0.25092	+0.20403	+0.15552
5.8.....	+0.35040	+0.30553	+0.25897	+0.21070	+0.16070
5.9.....	+0.36096	+0.31493	+0.26711	+0.21746	+0.16596
6.0.....	+0.37160	+0.32443	+0.27534	+0.22430	+0.17129
6.1.....	+0.38232	+0.33400	+0.28365	+0.23122	+0.17668
6.2.....	+0.39311	+0.34366	+0.29204	+0.23822	+0.18215
6.3.....	+0.40397	+0.35339	+0.30052	+0.24529	+0.18768
6.4.....	+0.41488	+0.36319	+0.30906	+0.25244	+0.19328
6.5.....	+0.42586	+0.37307	+0.31769	+0.25966	+0.19894
6.6.....	+0.43689	+0.38300	+0.32638	+0.26695	+0.20467
6.7.....	+0.44796	+0.39300	+0.33514	+0.27431	+0.21046
6.8.....	+0.45908	+0.40305	+0.34397	+0.28174	+0.21631
6.9.....	+0.47024	+0.41316	+0.35286	+0.28924	+0.22223
7.0.....	+0.48143	+0.42332	+0.36180	+0.29679	+0.22820
7.1.....	+0.49265	+0.43352	+0.37081	+0.30441	+0.23424
7.2.....	+0.50389	+0.44377	+0.37987	+0.31209	+0.24033
7.3.....	+0.51515	+0.45405	+0.38898	+0.31983	+0.24647
7.4.....	+0.52642	+0.46437	+0.39815	+0.32762	+0.25268
7.5.....	+0.53770	+0.47472	+0.40735	+0.33547	+0.25894
7.6.....	+0.54898	+0.48509	+0.41660	+0.34337	+0.26525
7.7.....	+0.56026	+0.49549	+0.42589	+0.35132	+0.27161
7.8.....	+0.57153	+0.50590	+0.43522	+0.35932	+0.27802
7.9.....	+0.58280	+0.51633	+0.44458	+0.36736	+0.28449
8.0.....	+0.59404	+0.52677	+0.45398	+0.37545	+0.29100
8.1.....	+0.60526	+0.53722	+0.46340	+0.38358	+0.29756
8.2.....	+0.61646	+0.54767	+0.47284	+0.39175	+0.30416
8.3.....	+0.62762	+0.55812	+0.48231	+0.39996	+0.31081
8.4.....	+0.63874	+0.56856	+0.49180	+0.40821	+0.31751
8.5.....	+0.64982	+0.57899	+0.50131	+0.41649	+0.32424
8.6.....	+0.66086	+0.58941	+0.51083	+0.42480	+0.33102
8.7.....	+0.67184	+0.59982	+0.52035	+0.43314	+0.33784
8.8.....	+0.68277	+0.61020	+0.52989	+0.44151	+0.34470
8.9.....	+0.69363	+0.62055	+0.53943	+0.44990	+0.35159
9.0.....	+0.70442	+0.63088	+0.54897	+0.45832	+0.35852
9.1.....	+0.71514	+0.64117	+0.55851	+0.46676	+0.36548
9.2.....	+0.72579	+0.65142	+0.56805	+0.47522	+0.37248
9.3.....	+0.73635	+0.66164	+0.57758	+0.48370	+0.37951
9.4.....	+0.74682	+0.67181	+0.58710	+0.49219	+0.38657
9.5.....	+0.75720	+0.68192	+0.59660	+0.50069	+0.39366
9.6.....	+0.76749	+0.69199	+0.60609	+0.50921	+0.40078
9.7.....	+0.77767	+0.70200	+0.61555	+0.51773	+0.40792
9.8.....	+0.78775	+0.71194	+0.62499	+0.52626	+0.41509
9.9.....	+0.79771	+0.72182	+0.63441	+0.53480	+0.42229
10.0.....	+0.80756	+0.73164	+0.64380	+0.54333	+0.42950

TABLE 6—Continued

$r$	$k^2=1.75$	$k^2=1.50$	$k^2=1.25$	$k^2=1.00$	$k^2=0.80$	$k^2=0.70$
0.015						
2596	-0.62614	-0.99935	-0.39619	+0.70992	+0.96794	+0.65901
3070	-0.40081	-0.99126	-0.58939	+0.55597	+1.00096	+0.77498
3551	-0.14789	-0.92475	-0.75369	+0.38028	+1.00280	+0.86986
4040	+0.11521	-0.80371	-0.88101	+0.18971	+0.97337	+0.94108
4537	+0.37037	-0.63525	-0.96509	-0.00829	+0.91357	+0.98667
5041	+0.60000	-0.42930	-1.00177	-0.20597	+0.82523	+1.00538
5552	+0.78828	-0.19800	-0.98923	-0.39557	+0.71110	+0.99666
6070	+0.92218	+0.04500	-0.92805	-0.56964	+0.57473	+0.96074
6596	+0.99248	+0.28533	-0.82124	-0.72136	+0.42037	+0.89857
7129	+0.99428	+0.50880	-0.67401	-0.84474	+0.25284	+0.81182
7668	+0.92745	+0.70218	-0.49361	-0.93493	+0.07738	+0.70286
8215	+0.79659	+0.85402	-0.28891	-0.98836	-0.10050	+0.57466
8768	+0.61071	+0.95533	-0.06997	-1.00292	+0.27522	+0.43071
9328	+0.38265	+1.00010	+0.15241	-0.97801	-0.44131	+0.27496
9894	+0.12815	+0.98566	+0.36728	-0.91460	-0.59353	+0.11166
10467	+0.13521	+0.91285	+0.56404	-0.81517	-0.72709	+0.05470
10984	+0.38922	+0.78597	+0.73299	-0.68362	-0.83780	-0.21956
11631	+0.61632	+0.61253	+0.86576	-0.52514	-0.92214	-0.37839
12223	+0.80080	+0.40279	+0.95581	-0.34595	-0.97747	-0.52681
12820	+0.92989	+0.16917	+0.99868	-0.15312	-1.00202	-0.66075
13424	+0.99465	-0.07448	+0.99224	+0.04575	-0.99501	-0.77650
14033	+0.99059	-0.31371	+0.93679	+0.24282	-0.95665	-0.87088
14647	-0.91798	-0.53433	+0.83507	+0.43030	-0.88813	-0.94127
15268	-0.78183	-0.72326	+0.69208	+0.60079	-0.79162	-0.98573
15894	-0.59157	-0.86927	+0.51490	+0.74755	-0.67013	-1.00302
16525	-0.36035	-0.96371	+0.31226	-0.86478	-0.52750	-0.99266
17161	-0.10417	-1.00095	+0.09418	+0.94784	-0.36822	-0.95492
17802	+0.15922	-0.97877	-0.12855	+0.99344	-0.19732	-0.89083
18449	+0.41157	-0.89848	-0.34492	+0.99976	-0.02018	-0.80216
19100	+0.63542	-0.76485	-0.54423	+0.96656	+0.15759	-0.69134
19756	+0.81525	-0.58580	-0.71662	+0.89512	+0.33038	-0.56143
20416	+0.93860	-0.37196	-0.85354	+0.78828	+0.49273	-0.41601
21081	+0.99690	-0.13602	-0.94822	+0.65026	+0.63951	-0.25909
21756	+0.98612	+0.10800	-0.99597	+0.48650	+0.76606	-0.09502
22424	+0.90700	+0.34560	-0.99442	+0.30348	+0.86839	+0.07168
23100	+0.76501	+0.56267	-0.94362	+0.10845	+0.94324	+0.23640
23782	+0.56998	+0.74629	-0.84611	-0.09087	+0.98825	+0.39458
24469	+0.33545	+0.88557	-0.70669	-0.28659	+1.00199	+0.54185
25156	+0.07765	+0.97220	-0.53226	-0.47096	+0.98402	+0.67412
25849	-0.18553	+1.00104	-0.33147	-0.63668	+0.93489	+0.78773
26546	-0.43584	+0.97037	-0.11426	-0.77717	+0.85616	+0.87953
27249	-0.65593	+0.88200	+0.10861	-0.88687	+0.75031	+0.94698
27956	-0.83052	+0.74119	+0.32610	-0.96142	+0.62070	+0.98821
28669	-0.94752	+0.55630	+0.52743	-0.99788	+0.47143	+1.00206
29381	-0.99878	+0.33833	+0.70262	-0.99478	+0.30721	+0.98815
30094	-0.98076	+0.10023	+0.84298	-0.95224	+0.13327	+0.94686
30807	-0.89470	-0.14383	+0.94155	-0.87195	-0.04490	+0.87933
31524	-0.74657	-0.37933	+0.99344	-0.75709	-0.22165	+0.78742
32246	-0.54664	-0.59227	+0.99607	-0.61220	-0.39137	+0.67368
32969	-0.30877	-0.76997	+0.94932	-0.44303	-0.54868	+0.54127
33694	-0.04948	-0.90185	+0.85548	-0.25629	-0.68859	+0.39384
34416	+0.21324	-0.98007	+0.71922	-0.05939	-0.80666	+0.23549
35149	+0.46116	-0.99998	+0.54729	+0.13988	-0.89915	+0.07061
35881	+0.67708	-0.96037	+0.34821	+0.33359	-0.96310	-0.09623
36616	+0.84600	-0.86360	+0.13186	+0.51406	-0.99651	-0.26040
37349	+0.95621	-0.71544	-0.09103	+0.67414	-0.99829	-0.41734
38081	+1.00004	-0.52469	-0.30941	+0.80746	-0.96839	-0.56271
38816	+0.97445	-0.30271	-0.51243	+0.90873	-0.90776	-0.69245
39549	+0.88121	-0.06271	-0.69003	+0.97394	-0.81831	-0.80297
40281	+0.72679	+0.18102	-0.83339	+1.00048	-0.70289	-0.89119
41016	+0.52192	+0.41398	-0.93539	+0.98730	-0.56515	-0.95467
41749	+0.28081	+0.62229	-0.99097	+0.93492	-0.40947	-0.99164
42481	+0.02020	+0.79354	-0.99736	+0.84542	-0.24079	-1.00106
43216	-0.24181	+0.91755	-0.95425	+0.72236	-0.06446	-0.98269
43949	-0.48703	+0.98692	-0.86378	+0.57060	+0.11392	-0.93701
44681	-0.69843	+0.99752	-0.73043	+0.39619	+0.28867	-0.86530
45416	-0.86133	+0.94871	-0.56081	+0.20604	+0.45426	-0.76955
46149	-0.96441	+0.84341	-0.36336	+0.00770	+0.60542	-0.65242
46881	-1.00051	+0.68787	-0.14787	-0.19094	+0.73735	-0.51715
47616	-0.96712	+0.49137	+0.07497	-0.38199	+0.84584	-0.36752
48349	-0.86657	+0.26559	+0.29408	-0.55787	+0.92747	-0.20767
49081	-0.70582	+0.02400	+0.49859	-0.71159	+0.97961	-0.04205
49816	-0.49605	-0.21902	+0.67835	-0.83703	+1.00063	+0.12475
50549	-0.25182	-0.44899	+0.82441	-0.92922	+0.98985	+0.28807
51281	+0.00990	-0.65222	+0.92954	-0.98448	+0.94761	+0.44338
52016	+0.27093	-0.81659	+0.98850	-1.00062	+0.87524	+0.58637

TABLE 6—Continued

$r$	$k^2=0.60$	$k^2=0.50$	$k^2=0.45$	$k^2=0.35$	$k^2=0.25$	$k^2=0.20$
10.0	+0.09143	-0.55689	-0.81987	-1.00115	-0.49878	+0.00070
10.2	+0.24276	-0.43489	-0.73504	-1.01259	-0.58150	-0.08648
10.4	+0.38846	-0.30454	-0.63756	-1.01065	-0.65886	-0.17304
10.6	+0.52515	-0.16833	-0.52910	-0.99532	-0.73011	-0.25835
10.8	+0.64962	-0.02889	-0.41150	-0.96678	-0.79460	-0.34177
11.0	+0.75898	+0.11112	-0.28679	-0.92539	-0.85168	-0.42269
11.2	+0.85066	+0.24898	-0.15712	-0.87166	-0.90082	-0.50052
11.4	+0.92250	+0.38202	-0.02472	-0.80630	-0.94153	-0.57466
11.6	+0.97282	+0.50767	+0.10811	-0.73016	-0.97342	-0.64456
11.8	+1.00041	+0.62349	+0.23906	-0.64424	-0.99615	-0.70968
12.0	+1.00461	+0.72721	+0.36585	-0.54969	-1.00950	-0.76955
12.2	+0.98530	+0.81681	+0.48626	-0.44775	-1.01333	-0.82368
12.4	+0.94292	+0.89055	+0.59820	-0.33979	-1.00758	-0.87168
12.6	+0.87844	+0.94696	+0.69969	-0.22726	-0.99228	-0.91315
12.8	+0.79337	+0.98494	+0.78896	-0.11166	-0.96756	-0.94779
13.0	+0.68968	+1.00374	+0.86444	+0.00544	-0.93365	-0.97531
13.2	+0.56979	+1.00296	+0.92478	+0.12247	-0.89086	-0.99550
13.4	+0.43652	+0.98263	+0.96893	+0.23785	-0.83957	-1.00818
13.6	+0.29299	+0.94310	+0.99609	+0.35000	-0.78026	-1.01325
13.8	+0.14256	+0.88516	+1.00578	+0.45741	-0.71350	-1.01066
14.0	-0.01122	+0.80991	+0.99782	+0.55861	-0.63991	-1.00041
14.2	-0.16474	+0.71882	+0.97233	+0.65223	-0.56018	-0.98258
14.4	-0.31437	+0.61366	+0.92974	+0.73699	-0.47508	-0.95727
14.6	-0.45659	+0.49648	+0.87080	+0.81172	-0.38541	-0.92469
14.8	-0.58805	+0.36958	+0.79654	+0.87541	-0.29204	-0.88505
15.0	-0.70564	+0.23543	+0.70825	+0.92718	-0.19586	-0.83867
15.2	-0.80658	+0.09667	+0.60749	+0.96631	-0.09779	-0.78588
15.4	-0.88847	-0.04399	+0.49601	+0.99226	+0.00122	-0.72707
15.6	-0.94939	-0.18379	+0.37579	+1.00467	+0.10022	-0.66270
15.8	-0.98787	-0.31998	+0.24894	+1.00336	+0.19826	-0.59325
16.0	-1.00301	-0.44988	+0.11769	+0.98834	+0.29437	-0.51924
16.2	-0.99443	-0.57093	+0.01564	+0.95980	+0.38764	-0.44125
16.4	-0.96234	-0.68077	-0.14869	+0.91814	+0.47715	-0.35986
16.6	-0.90749	-0.77721	-0.27911	+0.86391	+0.56203	-0.27571
16.8	-0.83116	-0.85835	-0.40460	+0.79785	+0.64146	-0.18943
17.0	-0.73515	-0.92261	-0.52292	+0.72086	+0.71466	-0.10168
17.2	-0.62175	-0.96869	-0.63200	+0.63399	+0.78093	-0.01316
17.4	-0.49361	-0.99569	-0.72988	+0.53843	+0.83959	+0.07547
17.6	-0.35379	-1.00308	-0.81484	+0.43547	+0.89009	+0.16351
17.8	-0.20558	-0.99069	-0.88537	+0.32654	+0.93193	+0.25029
18.0	-0.05250	-0.95878	-0.94020	+0.21313	+0.96469	+0.33513
18.2	+0.10183	-0.90795	-0.97837	+0.09679	+0.98805	+0.41738
18.4	+0.25374	-0.83922	-0.99919	-0.02088	+1.00178	+0.49639
18.6	+0.39963	-0.75393	-1.00228	-0.13827	+1.00573	+0.57154
18.8	+0.53604	-0.65375	-0.98760	-0.25375	+0.99986	+0.64226
19.0	+0.65974	-0.54068	-0.95538	-0.36574	+0.98423	+0.70799
19.2	+0.76778	-0.41692	-0.90621	-0.47270	+0.95899	+0.76822
19.4	+0.85759	-0.28493	-0.84094	-0.57315	+0.92437	+0.82247
19.6	+0.92705	-0.14732	-0.76074	-0.66571	+0.88071	+0.87032
19.8	+0.97449	-0.00679	-0.66702	-0.74909	+0.82843	+0.91140
20.0	+0.99879	+0.13387	-0.56145	-0.82215	+0.76805	+0.94537
20.2	+0.99936	+0.27189	-0.44591	-0.88388	+0.70014	+0.97197
20.4	+0.97619	+0.40453	-0.32243	-0.93343	+0.62538	+0.99100
20.6	+0.92983	+0.52917	-0.19322	-0.97010	+0.54449	+1.00229
20.8	+0.86137	+0.64334	-0.06058	-0.99338	+0.45826	+1.00575
21.0	+0.77244	+0.74479	+0.07314	-1.00296	+0.36753	+1.00137
21.2	+0.66515	+0.83150	+0.20556	-0.99870	+0.27320	+0.98915
21.4	+0.54204	+0.90175	+0.33433	-0.98064	+0.17619	+0.96921
21.6	+0.40605	+0.95416	+0.45714	-0.94905	+0.07745	+0.94169
21.8	+0.26039	+0.98768	+0.57181	-0.90434	-0.02205	+0.90680
22.0	+0.10855	+1.00164	+0.67630	-0.84715	-0.12133	+0.86481
22.2	+0.04588	+0.99577	+0.76875	-0.77824	-0.21943	+0.81605
22.4	-0.19922	+0.97018	+0.84751	-0.69858	-0.31536	+0.76090
22.6	-0.34781	+0.92537	+0.91117	-0.60926	-0.40820	+0.69979
22.8	-0.48813	+0.86223	+0.95859	-0.51152	-0.49702	+0.63191
23.0	-0.61683	+0.78200	+0.98893	-0.40670	-0.58095	+0.56163
23.2	-0.73085	+0.68628	+1.00165	-0.29626	-0.65916	+0.48566
23.4	-0.82747	+0.57696	+0.99651	-0.18171	-0.73088	+0.40588
23.6	-0.90440	+0.45620	+0.97361	-0.06466	-0.79541	+0.32292
23.8	-0.95979	+0.32640	+0.93335	+0.05329	-0.85211	+0.23742
24.0	-0.99233	+0.19042	+0.87645	+0.17051	-0.90041	+0.15005
24.2	-1.00123	+0.05067	+0.80391	+0.28536	-0.93984	+0.06151
24.4	-0.98629	-0.09009	+0.71703	+0.39626	-0.97001	-0.02751
24.6	-0.94786	-0.22907	+0.61737	+0.50168	-0.99061	-0.11632
24.8	-0.88685	-0.36350	+0.50669	+0.60014	-1.00145	-0.20422
25.0	-0.80471	-0.49071	+0.38697	+0.69029	-1.00241	-0.29051

TABLE 6—Continued

$r$	$k^2=0.175$	$k^2=0.150$	$k^2=0.125$	$k^2=0.10$	$k^2=0.09$	$k^2=0.07$
0.0070	+0.28215	+0.56022	+0.80805	+0.99063	+1.03575	+1.06005
0.0648	+0.20280	+0.49511	+0.76240	+0.96745	+1.02166	+1.06274
1.7304	+0.12219	+0.42741	+0.71352	+0.94114	+1.00468	+1.06327
25835	+0.04082	+0.35746	+0.66161	+0.91176	+0.98482	+1.06161
34177	-0.04081	+0.28563	+0.60686	+0.87939	+0.96213	+1.05774
42269	-0.12219	+0.21227	+0.54949	+0.84410	+0.93662	+1.05163
50052	-0.20279	+0.13779	+0.48974	+0.80600	+0.90837	+1.04327
57466	-0.28210	+0.06257	+0.42785	+0.76518	+0.87742	+1.03266
64456	-0.35961	-0.01299	+0.36409	+0.72178	+0.84386	+1.01979
70968	-0.43481	-0.08848	+0.29872	+0.67592	+0.80777	+1.00468
76955	-0.50723	-0.16349	+0.23204	+0.62775	+0.76923	+0.98733
82368	-0.57638	-0.23761	+0.16432	+0.57741	+0.72835	+0.96776
87168	-0.64180	-0.31043	+0.09587	+0.52508	+0.68524	+0.94600
91315	-0.70307	-0.38156	+0.02700	+0.47092	+0.64002	+0.92209
94779	-0.75977	-0.45058	-0.04200	+0.41511	+0.59282	+0.89606
97531	-0.81153	-0.51712	-0.11081	+0.35785	+0.54377	+0.86796
99550	-0.85799	-0.58081	-0.17912	+0.29932	+0.49303	+0.83784
100818	-0.89885	-0.64128	-0.24662	+0.23974	+0.44073	+0.80576
101325	-0.93381	-0.69819	-0.31299	+0.17930	+0.38705	+0.77179
101066	-0.96265	-0.75121	-0.37793	+0.11823	+0.33214	+0.73599
100041	-0.98516	-0.80004	-0.44114	+0.05673	+0.27618	+0.69843
98258	-1.00117	-0.84440	-0.50233	-0.00497	+0.21934	+0.65921
9727	-1.01058	-0.88404	-0.56121	-0.06666	+0.16179	+0.61840
2469	-1.01331	-0.91871	-0.61749	-0.12810	+0.10372	+0.57610
8505	-1.00933	-0.94821	-0.67093	-0.18908	+0.04532	+0.53240
3867	-0.99866	-0.97238	-0.72125	-0.24937	-0.01323	+0.48741
8588	-0.98135	-0.99106	-0.76823	-0.30875	-0.07173	+0.44123
2707	-0.95752	-1.00414	-0.81163	-0.36701	-0.13000	+0.39396
6270	-0.92731	-1.01154	-0.85125	-0.42392	-0.18785	+0.34572
9325	-0.89092	-1.01320	-0.88689	-0.47927	-0.24508	+0.29663
1924	-0.84858	-1.00911	-0.91839	-0.53286	-0.30151	+0.24681
4125	-0.80057	-0.99928	-0.94558	-0.58449	-0.35695	+0.19637
5986	-0.74721	-0.98377	-0.96833	-0.63396	-0.41121	+0.14544
7571	-0.68883	-0.96265	-0.98654	-0.68108	-0.46412	+0.09415
8943	-0.62584	-0.93603	-1.00009	-0.72568	-0.51549	+0.04262
10168	-0.55865	-0.90407	-1.00894	-0.76758	-0.56515	-0.00902
1316	-0.48771	-0.86693	-1.01302	-0.80662	-0.61293	-0.06063
7547	-0.41348	-0.82483	-1.01231	-0.84266	-0.65867	-0.11209
5351	-0.33647	-0.77800	-1.00680	-0.87554	-0.70221	-0.16327
5029	-0.25719	-0.72671	-0.99653	-0.90516	-0.74341	-0.21403
3513	-0.17618	-0.67124	-0.98153	-0.93138	-0.78213	-0.26425
1738	-0.09398	-0.61191	-0.96186	-0.95411	-0.81821	-0.31379
9639	-0.01115	-0.54906	-0.93762	-0.97325	-0.85156	-0.36253
1154	+0.07176	-0.48305	-0.90891	-0.98873	-0.88204	-0.41034
1226	+0.15419	-0.41425	-0.87587	-1.00049	-0.90954	-0.45709
9799	+0.23557	-0.34306	-0.83865	-1.00847	-0.93398	-0.50267
8822	+0.31535	-0.26989	-0.79743	-1.01264	-0.95527	-0.54695
2247	+0.39300	-0.19516	-0.75239	-1.01298	-0.97332	-0.58982
932	+0.46798	-0.11921	-0.70375	-1.00949	-0.98808	-0.63117
140	+0.53978	-0.04275	-0.65175	-1.00217	-0.99949	-0.67088
537	+0.60792	+0.03405	-0.59661	-0.99105	-1.00751	-0.70885
197	+0.67193	+0.11065	-0.53862	-0.97616	-1.01210	-0.74497
100	+0.73136	+0.18660	-0.47804	-0.95756	-1.01324	-0.77916
229	+0.78583	+0.26148	-0.41516	-0.93532	-1.01094	-0.81131
575	+0.83494	+0.33483	-0.35028	-0.90951	-1.00519	-0.84135
137	+0.87836	+0.40624	-0.28372	-0.88024	-0.99600	-0.86919
915	+0.91580	+0.47529	-0.21579	-0.84760	-0.98341	-0.89475
921	+0.94699	+0.54157	-0.14682	-0.81173	-0.96746	-0.91796
169	+0.60470	+0.60470	-0.07715	-0.77275	-0.94819	-0.93877
680	+0.97173	+0.66430	-0.00710	-0.73081	-0.92568	-0.95711
481	+0.98983	+0.66430	+0.06299	-0.68607	-0.89999	-0.97293
605	+1.00118	+0.72004	+0.13276	-0.63871	-0.87120	-0.98619
990	+1.00568	+0.77157	+0.20190	-0.58889	-0.83943	-0.99685
979	+1.00331	+0.81860	+0.27006	-0.53681	-0.80476	-1.00487
319	+0.99409	+0.86085	+0.33691	-0.48267	-0.76733	-1.01024
163	+0.97806	+0.89808	+0.40213	-0.42668	-0.72725	-1.01294
566	+0.95535	+0.93006	+0.46540	-0.36904	-0.68467	-1.01295
588	+0.92609	+0.95660	+0.52642	-0.30998	-0.63972	-1.01028
292	+0.89049	+0.97754	+0.58488	-0.24973	-0.59257	-1.00493
742	+0.84879	+0.99277	+0.64050	-0.18852	-0.54336	-0.99691
905	+0.80128	+1.00218	+0.69301	-0.12658	-0.49228	-0.98625
151	+0.74827	+1.00573	+0.74215	-0.06414	-0.43950	-0.97295
751	+0.69013	+1.00338	+0.78768	-0.00147	-0.38519	-0.95707
332	+0.62726	+0.99516	+0.82938	+0.06122	-0.32955	-0.93863
222	+0.56009	+0.98109	+0.86705	+0.12367	-0.27276	-0.91770
951	+0.48907	+0.96128	+0.90050	+0.18563	-0.21503	-0.89431
25.0	+0.41469	+0.93583				

TABLE 6—Continued

$r$	$k^2=0.055$	$k^2=0.05$	$k^2=0.045$	$k^2=0.035$	$k^2=0.02$
10.0	+1.00547	+0.97058	+0.92638	+0.80756	+0.54333
10.2	+1.01807	+0.98568	+0.94348	+0.82688	+0.56041
10.4	+1.02921	+0.99957	+0.95961	+0.84569	+0.57746
10.6	+1.03885	+1.01220	+0.97473	+0.86393	+0.59448
10.8	+1.04694	+1.02353	+0.98878	+0.88159	+0.61145
11.0	+1.05345	+1.03351	+1.00173	+0.89861	+0.62835
11.2	+1.05834	+1.04211	+1.01355	+0.91497	+0.64516
11.4	+1.06157	+1.04929	+1.02418	+0.93063	+0.66186
11.6	+1.06313	+1.05501	+1.03360	+0.94556	+0.67844
11.8	+1.06297	+1.05926	+1.04178	+0.95973	+0.69489
12.0	+1.06109	+1.06199	+1.04868	+0.97311	+0.71118
12.2	+1.05747	+1.06319	+1.05427	+0.98566	+0.72729
12.4	+1.05209	+1.06284	+1.05854	+0.99737	+0.74321
12.6	+1.04494	+1.06091	+1.06144	+1.00820	+0.75893
12.8	+1.03602	+1.05740	+1.06298	+1.01812	+0.77442
13.0	+1.02532	+1.05229	+1.06312	+1.02712	+0.78967
13.2	+1.01286	+1.04557	+1.06185	+1.03517	+0.80466
13.4	+0.99863	+1.03724	+1.05915	+1.04224	+0.81938
13.6	+0.98265	+1.02730	+1.05502	+1.04831	+0.83380
13.8	+0.96493	+1.01575	+1.04945	+1.05338	+0.84792
14.0	+0.94550	+1.00259	+1.04243	+1.05741	+0.86172
14.2	+0.92438	+0.98784	+1.03396	+1.06039	+0.87518
14.4	+0.90158	+0.97151	+1.02404	+1.06231	+0.88829
14.6	+0.87715	+0.95361	+1.01267	+1.06315	+0.90103
14.8	+0.85113	+0.93415	+0.99986	+1.06290	+0.91338
15.0	+0.82354	+0.91318	+0.98562	+1.06155	+0.92534
15.2	+0.79443	+0.89070	+0.96995	+1.05910	+0.93689
15.4	+0.76385	+0.86674	+0.95287	+1.05553	+0.94801
15.6	+0.73184	+0.84135	+0.93440	+1.05083	+0.95870
15.8	+0.69847	+0.81455	+0.91455	+1.04501	+0.96893
16.0	+0.66378	+0.78639	+0.89335	+1.03807	+0.97870
16.2	+0.62785	+0.75689	+0.87082	+1.02999	+0.98799
16.4	+0.59072	+0.72612	+0.84699	+1.02079	+0.99679
16.6	+0.55247	+0.69410	+0.82189	+1.01046	+1.00509
16.8	+0.51316	+0.66090	+0.79555	+0.99901	+1.01288
17.0	+0.47287	+0.62657	+0.76800	+0.98644	+1.02015
17.2	+0.43167	+0.59116	+0.73928	+0.97277	+1.02688
17.4	+0.38964	+0.55472	+0.70943	+0.95799	+1.03307
17.6	+0.34686	+0.51733	+0.67849	+0.94213	+1.03870
17.8	+0.30340	+0.47903	+0.64651	+0.92520	+1.04378
18.0	+0.25935	+0.43989	+0.61352	+0.90720	+1.04828
18.2	+0.21479	+0.39999	+0.57958	+0.88816	+1.05220
18.4	+0.16982	+0.35938	+0.54474	+0.86808	+1.05553
18.6	+0.12451	+0.31813	+0.50905	+0.84700	+1.05827
18.8	+0.07895	+0.27633	+0.47256	+0.82493	+1.06041
19.0	+0.03324	+0.23403	+0.43533	+0.80189	+1.06193
19.2	-0.01253	+0.19132	+0.39740	+0.77791	+1.06285
19.4	-0.05829	+0.14827	+0.35885	+0.75300	+1.06314
19.6	-0.10392	+0.10495	+0.31973	+0.72720	+1.06281
19.8	-0.14935	+0.06144	+0.28011	+0.70054	+1.06185
20.0	-0.19448	+0.01782	+0.24003	+0.67303	+1.06026
20.2	-0.23922	-0.02583	+0.19957	+0.64472	+1.05803
20.4	-0.28348	-0.06943	+0.15879	+0.61564	+1.05517
20.6	-0.32718	-0.11291	+0.11776	+0.58581	+1.05166
20.8	-0.37021	-0.15619	+0.07653	+0.55527	+1.04751
21.0	-0.41250	-0.19918	+0.03518	+0.52405	+1.04271
21.2	-0.45396	-0.24181	-0.00622	+0.49220	+1.03727
21.4	-0.49450	-0.28399	-0.04762	+0.45974	+1.03119
21.6	-0.53403	-0.32566	-0.08894	+0.42673	+1.02446
21.8	-0.57249	-0.36674	-0.13011	+0.39318	+1.01708
22.0	-0.60978	-0.40714	-0.17107	+0.35916	+1.00907
22.2	-0.64583	-0.44680	-0.21175	+0.32469	+1.00041
22.4	-0.68057	-0.48563	-0.25208	+0.28982	+0.99112
22.6	-0.71391	-0.52357	-0.29201	+0.25459	+0.98119
22.8	-0.74580	-0.56055	-0.33145	+0.21904	+0.97063
23.0	-0.77616	-0.59649	-0.37034	+0.18322	+0.95944
23.2	-0.80494	-0.63133	-0.40863	+0.14717	+0.94763
23.4	-0.83206	-0.66500	-0.44624	+0.11094	+0.93520
23.6	-0.85747	-0.69744	-0.48311	+0.07457	+0.92216
23.8	-0.88112	-0.72858	-0.51918	+0.03810	+0.90852
24.0	-0.90296	-0.75837	-0.55439	+0.00159	+0.89427
24.2	-0.92293	-0.78675	-0.58868	-0.03493	+0.87944
24.4	-0.94100	-0.81366	-0.62199	-0.07140	+0.86402
24.6	-0.95713	-0.83906	-0.65427	-0.10778	+0.84803
24.8	-0.97128	-0.86288	-0.68545	-0.14403	+0.83147
25.0	-0.98342	-0.88510	-0.71549	-0.18009	+0.81436

# SPECTROGRAPHIC OBSERVATIONS OF THE ECLIPSING VARIABLE TT HYDRAE\*

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## ABSTRACT

The spectrum of TT Hydrae during and near total phase of principal eclipse shows double emission lines of  $H$  which undergo eclipses similar to those of RW Tauri, SX Cassiopeiae, RX Cassiopeiae, etc. This fact and the asymmetry of the velocity-curve from the  $H$  lines suggest that the picture described by Struve for SX Cas holds also for TT Hya.

## PHOTOMETRIC DATA

TT Hydrae<sup>1</sup> was discovered to be an eclipsing variable of the Algol type by H. E. Wood<sup>2</sup> in 1926; the provisional designation was 28.1926. At the time of the announcement Wood published a light-curve and gave an approximate period of 6.96 days. The range in photographic magnitude is from 7.6 to 10.2; no secondary minimum was observed.

The only other more accurate determination of the period belongs to Hertzsprung,<sup>3</sup> who based his conclusions upon the study of Harvard photographs. His values for the epoch of minimum and the period are

$$\text{JD } 2424615.388 + 6.953401 \text{ E.}$$

Approximate photometric solutions were undertaken by M. Shapley,<sup>4</sup> who found:

Range of secondary (computed)....	0.01 mag.	
	Uniform Solution	Darkened Solution
$t'$ .....	0.33 day	0.32 day
$t''$ .....	0.12 day	0.12 day
$M_1$ .....	+ 3.6 mag.	+ 3.7 mag.
$M_2$ .....	+ 1.3 mag.	+ 1.4 mag.
$m_1$ .....	10.06 mag.	10.06 mag.
$m_2$ .....	7.73 mag.	7.73 mag.
$L_1$ .....	0.105	0.105
$L_2$ .....	0.895	0.895
$J_2/J_1$ .....	95	95
$a_0$ .....	1.00	1.00
$i$ .....	82°5	82°6
$r_1$ .....	0.246	0.240
$r_2$ .....	0.074	0.072
$A$ .....	19.27	19.27
$\pi$ .....	0'005	0'005
$\rho_1$ .....	0.005	0.005
$\rho_2$ .....	0.50	0.55

The principal eclipse lasts 13 hours, and the total phase lasts 5.5 hours.<sup>5</sup>

\* Contributions from the McDonald Observatory, University of Texas, No. 120.

<sup>1</sup> CD - 25°8531 (8.0 mag.) ( $\alpha = 11^h 7^m 1$ ;  $\delta = -25^\circ 46' 9$  [1875.0]) = CPD - 25°4711 (7.4 mag.) = Gou 15354 (8 mag.) = Cord A 8686 (8.1 mag.) = HD 97528 (Sp. A2).

<sup>2</sup> Union Obs. Circ., No. 69, 1926; Beob. Zirk., 8, 50, 1926; A.N., 228, 353, 1926.

<sup>3</sup> Union Obs. Circ., No. 77, 1928; B.A.N., 4, 153, 1928.

<sup>4</sup> Harvard Bull., No. 847, 1927.

<sup>5</sup> Kl. Veröff. Berlin-Babelsberg, No. 22, p. 203, 1940.

## THE PROBLEM

In his paper on the spectra of eclipsing binaries Wyse<sup>6</sup> reported that the spectrum of the secondary star of the system TT Hya "contains bright hydrogen lines." Wyse's remark was based upon two plates taken during eclipse, which he described in the following terms:

On the plate taken JD 2427521.873 [at phase 6.917 days, according to Hertzsprung's epoch and period],  $H\beta$  is very weak both in emission and absorption, the emission line appearing on the less refrangible edge of the absorption line. The same is true of  $H\delta$ , except that the lines are stronger and clearer both in emission and absorption. The emission at  $H\gamma$  is very doubtful, and the absorption line is blended with  $\lambda$  4338 of  $Ti$  II. On the plate taken JD 2427542.805 [at phase 0.035 days],  $H\beta$  appears clearly as an emission line in the normal position; the spectrum is continuous at  $H\gamma$ , and  $H\delta$  is an absorption line.

Wyse finally stated that "observations at principal minimum should be continued." His estimate of the spectral type of the secondary component of TT Hya was dG6p.

Three years after Wyse's paper, Sanford<sup>7</sup> published a spectrographic orbit for TT Hya, based upon 16 spectrograms, all obtained outside of the principal eclipse. The orbit was sensibly circular, and the elements were the following:

$$\begin{array}{ll} P = 6.953401 \text{ days (assumed)} & T = \text{JD } 2424615.079 \text{ [phase 6.644 days]} \\ \gamma = + 11.4 \text{ km/sec} & a_1 \sin i = 3.07 \times 10^6 \text{ km} \\ K = 32.2 \text{ km/sec} & \frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = 0.024 \odot \\ e = 0.08 & \\ \omega = 64^\circ 0 & \end{array}$$

From Joy's observations of RW Tauri<sup>8</sup> at minimum light and Struve's spectrographic studies of the systems of SX Cassiopeiae<sup>9</sup> and RX Cassiopeiae,<sup>10</sup> it became evident that TT Hya deserved further spectrographic study. Consequently, the star was placed on the observing program of the 82-inch reflecting telescope of the McDonald Observatory.

## THE OBSERVATIONS

The observations were made principally during the months of February and March, 1944 (some of them by Dr. O. Struve and Dr. H. Steel), but it was not possible to obtain any spectrograms during or near the total phase of the principal eclipse. A few additional observations at maximum light were secured by Dr. Struve in April and May, 1944, and one by Dr. W. A. Hiltner in March, 1945. The last set of spectrograms was obtained by Dr. Struve and one of the writers (Sahade) in May and June, 1945. During this last period three exposures showing the spectrum of the secondary star were obtained; but, because of the period of TT Hya and the time at which principal minima occurred, they correspond to phases after mid-eclipse.

Most of the spectrograms were obtained on glass plates of Eastman 103a-O emulsion, with the combination of two quartz prisms and a 500-mm camera, which gives a dispersion of 55 Å/mm at  $H\gamma$ . The three exposures showing the spectrum of the secondary star were made on Eastman 103a-O film with two glass prisms and a 180-mm Schmidt camera, a combination which gives a dispersion of 76 Å/mm at  $H\gamma$ . Finally, the spectrogram secured by Dr. Hiltner was taken on Eastman 103a-E film with two quartz prisms and the 180-mm Schmidt camera. In the table of radial velocities, "CQ" indicates the plates, "G f/2" the films taken with the glass prisms, and "Q f/2" the film obtained with the quartz prisms.

<sup>6</sup> *Lick Obs. Bull.*, No. 464, 1934.

<sup>7</sup> *Ap. J.*, **86**, 153, 1937; *Mt. W. Contr.*, No. 574.

<sup>8</sup> *Pub. A.S.P.*, **54**, 35, 1942.

<sup>9</sup> *Ap. J.*, **99**, 89, 1944; *McD. Contr.*, No. 84.

<sup>10</sup> *Ap. J.*, **99**, 295, 1944; *McD. Contr.*, No. 94.

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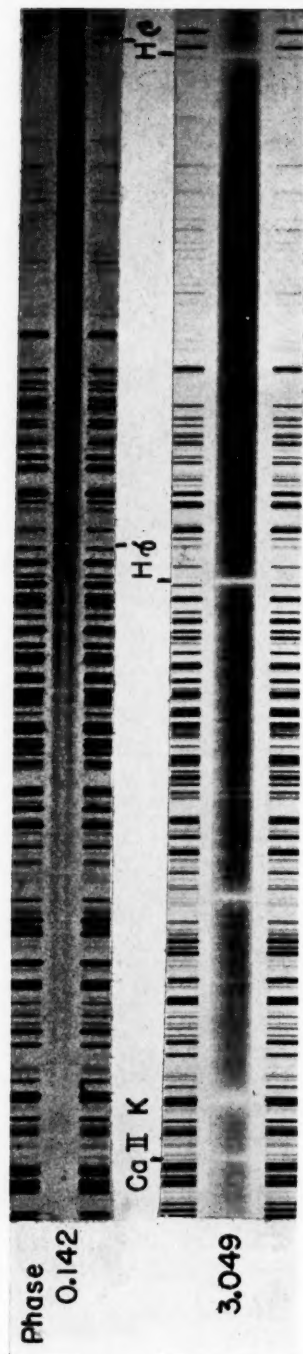
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PLATE V



SPECTRA OF TT HYDRAE

At maximum light the exposure time was of the order of 25 minutes, in fair seeing. The slit width was 0.075 mm for the plates and 0.10 mm for the films. The total number of spectrograms is 43.

## THE SPECTRUM (PL. V)

The plates taken at maximum light show a spectrum which corresponds to about type A3. On most of the plates the *H* lines are very strong and have sharp and deep cores with relatively not very pronounced Stark wings. There are lines of *Fe* I, *Fe* II, *Ti* II, *Ca* I,

TABLE 1  
RADIAL VELOCITIES OF TT HYDRAE

PLATE	DATE	U.T.	PHASE (IN DAYS)	NUM- BER OF MEAS- URES	RADIAL VELOCITIES IN KM/SEC FROM—	
					<i>Ca</i> II K	<i>H</i>
Gf/2 5712.....	1945 May 28	3:59	0.110	2	.....	+ 2.0
5721.....	June 4	3:38	0.142	2	.....	- 7.9
5713.....	May 28	5:00	0.152	2	.....	+ 2.1
CQ 2903.....	1944 Feb. 17	8:54	0.193	2	- 4.8	+ 7.1
2967.....	Mar. 2	7:49	0.241	2	- 9.6	-16.6
2904.....	Feb. 17	10:50	0.273	2	- 6.6	+ 9.6
Qf/2 5670.....	1945 Mar. 27	6:28	0.794	1	+47.9*	- 0.8
CQ 4485.....	June 5	2:48	1.107	2	+13.3	+ 7.6
2936.....	1944 Feb. 25	9:02	1.245	2	+10.2	- 4.2
3058.....	Mar. 17	6:59	1.258	1	+ 5.6	+ 3.3
2863.....	Feb. 11	12:16	1.286	2	.....	-11.4
3059.....	Mar. 17	7:29	1.320	1	-25.0	- 2.6
3008.....	10	9:35	1.361	2	+16.2	- 3.3
4441.....	1945 May 30	2:41	2.056	1	- 4.9	- 2.2
2870.....	1944 Feb. 12	9:04	2.154	2	-24.1	-43.7
2917.....	19	9:08	2.202	1	-40.3	-32.7
2941.....	26	8:18	2.214	2	- 9.8	-21.6
3015.....	Mar. 11	6:36	2.237	1	-10.0	-12.0
2974.....	4	8:22	2.264	1	- 9.8	-32.8
4449.....	1945 May 31	2:32	3.049	1	-26.5	- 7.0
4450.....	31	2:58	3.067	1	-18.9	-14.6
2922.....	1944 Feb. 20	8:09	3.161	2	-30.2	-13.0
2877.....	13	9:40	3.178	1	-13.8	-21.0
2982.....	Mar. 5	8:01	3.249	1	-10.3	-37.1
3024.....	12	7:24	3.270	1	- 7.3	-21.4
3065.....	Apr. 30	5:54	3.533	1	-31.0	-48.3
4415.....	1945 May 25	2:48	4.014	1	+30.4	- 7.4
2885.....	1944 Feb. 14	9:32	4.172	1	+19.2	-13.3
2987.....	Mar. 6	6:36	4.190	1	+13.7	- 1.8
2944.....	Feb. 28	8:27	4.220	2	+23.6	- 9.6
3034.....	Mar. 13	6:42	4.241	1	+25.7	+ 0.8
3035.....	13	7:16	4.264	2	+25.8	- 6.4
3083.....	May 1	2:33	4.394	1	+18.1	-12.6
4461.....	1945 June 2	2:35	5.051	1	+41.2	+17.0
4462.....	2	3:02	5.070	1	+59.5	+37.0
2892.....	1944 Feb. 15	8:37	5.134	2	+50.6	+33.8
2994.....	Mar. 7	7:53	5.244	2	+54.5	+62.3
2950.....	Feb. 29	9:11	5.251	4	+64.1	+70.8
3045.....	Mar. 14	7:51	5.289	2	+57.2	+61.6
3106.....	May 2	2:37	5.397	2	+56.4	+52.8
4468.....	1945 June 3	3:04	6.072	2	+44.2	+34.3
2958.....	1944 Mar. 1	6:28	6.138	2	+67.2	+65.4
3051.....	15	9:06	6.341	2	+60.6	+58.7

\* This value was disregarded.

$Sr II$ ;  $Mg II$  4481 and  $Si II$  are relatively weak; but  $Fe II$  is considerably stronger than  $Fe I$ .

The spectrum of the secondary star appears only during and near the total phase of eclipse; its spectral type, estimated from the ratio  $\lambda 4325/H\gamma$  on the film G f/2 5712 at phase 0.110 days (during total phase), is about G5. Although the spectrogram is not very well exposed beyond  $\lambda 4200$ , it seems that the line  $\lambda 4215$  is much stronger than it would appear in a dwarf star. The three spectrograms on which the spectrum of the

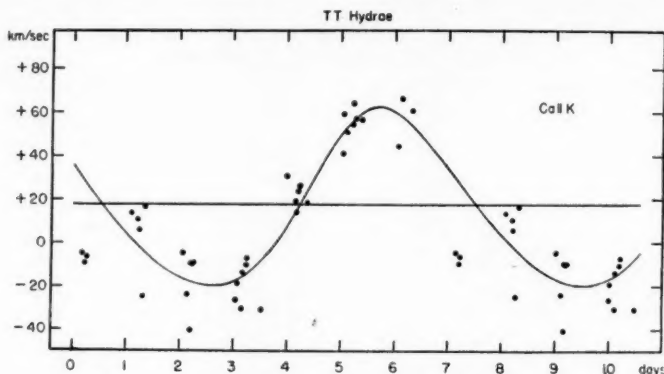


FIG. 1.—Velocity-curve of TT Hydrae from the  $Ca II$  K line

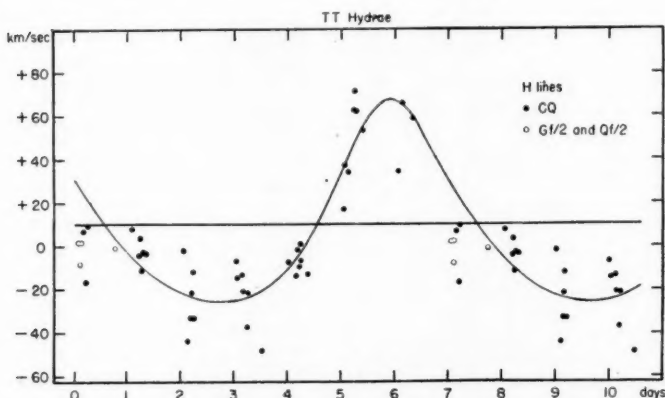


FIG. 2.—Velocity-curve of TT Hydrae from the  $H$  lines

secondary star is present (G f/2 5712, G f/2 5721, and G f/2 5713, at phases 0.110, 0.142, and 0.152 days, respectively) show a strong emission of  $H\gamma$  and  $H\beta$  on the violet side of the absorption line. It seems that at  $H\beta$  there is also a very weak red emission.

#### THE RADIAL VELOCITIES

On our spectrograms the only lines suitable for measurement are the K line of  $Ca II$ ,  $H\delta$  and  $H\gamma$ ; the metallic lines, though not especially faint, give less reliable results. Owing to the fact that the  $H$  lines appear in emission during and near totality of the primary eclipse, the radial velocities from the K line of  $Ca II$  and the means of the individual results from  $H\delta$  and  $H\gamma$  were plotted separately. The numerical values are shown in Table

1, and the plots in Figures 1 and 2. The velocity-curve obtained from the  $H$  lines is more asymmetrical than the velocity-curve obtained from the  $K$  line of  $Ca II$ . The method of Lehmann-Filhés was applied to each of the plots. The orbital elements from the  $Ca II$   $K$  line are as follows:

$$\begin{array}{ll} P = 6.953401 \text{ days (assumed)} & T = \text{phase 5.1 days} \\ \gamma = + 17.5 \text{ km/sec} & a \sin i = 3.9 \times 10^6 \text{ km} \\ K = 41.2 \text{ km/sec} & \frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = 0.05 \odot \\ e = 0.12 & \\ \omega = 319^\circ 8 & \end{array}$$

The orbital elements from the  $H$  lines are:

$$\begin{array}{ll} P = 6.953401 \text{ days (assumed)} & T = \text{phase 5.7 days} \\ \gamma = + 10.0 \text{ km/sec} & a \sin i = 4.3 \times 10^6 \text{ km} \\ K = 46.3 \text{ km/sec} & \frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = 0.07 \odot \\ e = 0.24 & \\ \omega = 342^\circ 2 & \end{array}$$

In computing graphically the orbital elements we have not considered the radial velocities from the plates taken during the eclipse, since they depart appreciably from the velocity-curve suggested by the rest of the measurements. These values are not necessarily interpreted as evidence of a rotational disturbance because at maximum light the lines are sharp. Nor can they be considered as arising from the secondary star—except on the three films already mentioned—since its spectrum is not shown at these phases.

The emission lines indicate velocities of the order of 270 km/sec.

#### CONCLUSIONS

The most interesting observational features of TT Hya are the emission lines of  $H$  which appear during and near totality of the principal eclipse. As far as the behavior of these emission lines during total eclipse is concerned, our observations, combined with those of Wyse already mentioned, allow us to state that we are dealing here with double emissions (as in the cases of RW Tau,<sup>8</sup> SX Cas,<sup>9</sup> RX Cas,<sup>10</sup> RW Per,<sup>11</sup> VW Cyg,<sup>12</sup> and AQ Peg<sup>13</sup>) which experience eclipses at principal minimum. In TT Hya just before mid-eclipse we see only the red bright component; just after mid-eclipse we see only the violet component; and at mid-eclipse the emission lines appear "in the normal position."

The emission lines are not seen when the star is at maximum light or at the partial phases of the eclipse. In this respect, TT Hya is more like RW Per, VW Cyg, and AQ Peg. The presence of the bright lines and their peculiar behavior suggest the existence of a gaseous ring around the A star. Furthermore, the velocity-curve from the  $K$  line of  $Ca II$  suggests that at about the phase corresponding to secondary minimum the  $H$  lines may be affected by a stream of gas moving away from the A star with negative velocity, which distorts the velocity-curve from these lines and flattens the region of maximum negative velocity. We conclude, then, that the picture described by Struve for SX Cas holds also in this case.

It is reasonable to think that the motion of the A star is represented by the velocity-curve from the  $K$  line of  $Ca II$ , but we must keep in mind that in the case of RX Cas, Struve concluded that probably the velocity-curve of the primary star "is subject to distortions which are produced in the gaseous stream." Therefore, the orbital elements derived from the  $K$  line of  $Ca II$  must be accepted with caution. It is unfortunate that the photometric observations do not give any indication concerning the eccentricity.

<sup>11</sup> O. Struve, *Ap. J.*, **102**, 89, 1945; *McDonald Contr.*, No. 110.

<sup>12</sup> O. Struve, *Harvard Announcement Card*, No. 715, August, 1945; *Ap. J.*, **103**, 87, 1946; *McDonald Contr.* No. 121.

<sup>13</sup> O. Struve, *Harvard Announcement Card*, No. 717, August, 1945; *Ap. J.*, **103**, 92, 1946; *McDonald Contr.* No. 121.

# SPECTROGRAPHIC OBSERVATIONS OF ELEVEN ECLIPSING BINARIES\*

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## ABSTRACT

The spectral characteristics, velocity-curves, and spectrographic orbital elements are given for SS Librae, UU Ophiuchi, W Scuti, RZ Draconis, U Scuti, QY Aquilae, VW Cygni, AQ Pegasi, XX Cephei, WW Andromedae, and XY Cephei. Two stars, VW Cyg and AQ Peg, have bright *H* lines during principal eclipse, and these undergo eclipses as in RW Persei and several other stars.

## THE OBSERVATIONS

This paper is a continuation of earlier work on the spectra and velocity-curves of eclipsing variables.<sup>1</sup> The principal purpose of the new series of observations was to find systems whose spectra have bright *H* lines during principal eclipse. The result of my earlier observations was that "there are few, if any, criteria to predict which stars are most likely to show these effects." Hence, the observing program (Table 1) was chosen

TABLE 1\*

THE OBSERVING PROGRAM

No.	Star	1945		PERI- OD (DAYS)	CHAR.	MAGNITUDE			<i>D</i>	<i>d</i>	Sp.	REMARKS	No. OF PLATES
		$\alpha$	$\delta$			<i>M</i>	<i>A</i> <sub>1</sub>	<i>A</i> <sub>2</sub>					
1....	SS Lib	15 <sup>h</sup> 46 <sup>m</sup> 0	-15°22'	1.44	A	10.4	0.9	0.4	6.7	1.1	A5	Sec.? <i>D?</i> <i>d?</i>	23
2....	UU Oph	16 54.0	-25 43	4.40	A	10.0	2.4	0	12.5	2.	A0	Deep; short total	23
3....	W Sct....	18 21.6	-13 41	10.27	A	9.5	0.7	.15	30.	0	F?	Giants. Sec.? Ell.?	43
4....	RZ Dra	18 22.5	+58 52	0.55	$\beta$	10.0	0.8	.2	.....	.....	Ap	Double <i>P?</i>	39
5....	U Sct	18 51.4	-12 40	0.95	$\beta$	9.7	1.0	.3	.....	.....	F0	Close pair; dense <i>A</i> Late tp? Close; graz- ing ecl.?	5
6....	OY Aql	20 06.9	+15 09	7.23	A	10.8	2.9	.....	26.	3.	.....	.....	23
7....	VW Cyg	20 13.1	+34 20	8.43	A	9.7	2.6	0	24.	8.	A0	Deep total; ranges?	34
8....	AQ Peg	21 34.7	+13 12	5.55	A	10.0	2.7	.1	18.	5.	A2	Deep total; sec. displ.; <i>A</i> and <i>K</i>	37
9....	XX Cep	23 35.8	+64 02	2.34	A	8.7	0.8	.0	7.	0	A	.....	35
10....	WW And	23 42.0	+45 23	23.29	A	10.3	1.1	?	40.	7.	A5+ F3	Shal. tot?	29
11....	XY Cep	23 49.9	+68 38	2.77	A	10.0	0.8	0.0	10.	0	.....	Sec.? <i>d?</i>	28

\* The data for *M*, *A*<sub>1</sub>, *A*<sub>2</sub>, *D*, *d*, and Sp. are from H. Schneller, *Kat. u. Eph.* 1941 (*Kl. Veröff. Berlin-Babelsberg*, No. 22), except in the case of QY Aql, for which the data are those given by B. S. Whitney (*A.J.*, **50**, 131, 1943, and *A.p.J.*, **102**, 202, 1945). The remarks are from R. S. Dugan, *Contr. Princeton U. Obs.*, No. 15, 1934.

more or less at random from Dugan's *Finding List*, except for star No. 6, QY Aql, which was suggested to me by Professor B. S. Whitney of the University of Oklahoma. Stars No. 3, W Sct, and No. 10, WW And, were already on the McDonald program. All other stars were observed for the first time during the months of July and August, 1945. The instrument was the same as that used previously;<sup>2</sup> the glass prisms and the *f*/2 Schmidt camera were used exclusively in July and August, 1945. Some of the older spectrograms of W Sct were obtained with the quartz prisms and the 500-mm camera. The slit-width was usually set at 0.1 mm.

\* Contributions from the McDonald Observatory, University of Texas, No. 121.

<sup>1</sup> *A.p.J.*, **102**, 74, 1945.

<sup>2</sup> *Ibid.*, p. 89.

The photometric elements used for computing the phases are listed in Table 2. A more recent determination of the period of SS Lib by Soloviev<sup>3</sup> gives 1.438003 days. For WW And, Soloviev<sup>4</sup> found that three epochs observed by him agree well with the elements listed by Schneller in *Katalog und Ephemeriden veränderlicher Sterne* for 1940. During the months of July and August, 1945, Charles H. Whitney made numerous observations of the epochs of several stars, using for this purpose a 5-inch visual refractor

TABLE 2\*  
PHOTOMETRIC ELEMENTS

1.....	SS Lib	2420,251.470+ 1.43800 E
2.....	UU Oph	2418,833.505+ 4.396766 E
3.....	W Sct	2420,665.47 +10.2703 E
4.....	RZ Dra	2417,673.251+ 0.5508772 E
5.....	U Sct	2416,366.301+ 0.9549849 E
6.....	QY Aql	2430,223.615+ 7.229617 E
7.....	VW Cyg	2420,327.723+ 8.430274 E
8.....	AQ Peg	2424,749.335+ 5.54834 E
9.....	XX Cep	2425,096.430+ 2.337330 E
10.....	WW And	2422,719.40 +23.28556 E
11.....	XY Cep	2425,096.497+ 2.77455 E

\* The elements are from H. Schneller, *Kat. u. Eph.*, 1941 (*Kl. Veröff. Berlin-Babelsberg*, No. 22), except for QY Aql, for which unpublished data by B. S. Whitney have been used.

at the McDonald Observatory. I am indebted to him for the information contained in the accompanying tabulation. My own estimates at the 82-inch telescope indicate that

Star	No. of Estimates	Mid-eclipse (Minutes)	Star	No. of Estimates	Mid-eclipse (Minutes)
RZ Dra.....	105	4 min. early	XY Cep.....	29	3 min. early
XX Cep.....	36	20 late	VW Cyg.....	15	0 on time
SS Lib.....	31	15 early			

the minimum of AQ Peg occurred about 2 hours later than was predicted, and that of UU Oph occurred probably about one hour, or more, late. That of VW Cyg was about on time, as was also that of WW And.

Star No. 5, U Sct, was included only because I do not expect to be able to complete this orbit next year. In the meantime the few observations secured in 1945 will suffice to give a rough approximation of the range in radial velocity. Eight of the eleven stars have photometric orbits (Table 3). The other three, SS Lib, XX Cep, and XY Cep, have no orbits.

The star lines used in all eleven stars are listed in Table 4. Table 5 gives the radial velocities and Table 6 the spectrographic elements. The orbits of SS Lib, UU Oph, W Sct, RZ Dra, U Sct, WW And, and XY Cep are circular, and the corresponding velocity-curves exhibit no noticeable asymmetry. The velocity-curves of QY Aql and XX Cep are sufficiently unsymmetrical to warrant elliptical elements. The velocity-curves of VW Cyg and AQ Peg are peculiar and resemble those of RW Per<sup>5</sup> and SX Casiopeiae.<sup>6</sup> All four velocity-curves have broad and shallow minima between phases 0.3 P

<sup>3</sup> *Astr. Circ. Acad. Sci. Soviet Union*, No. 16, 1943.

<sup>5</sup> Struve, *op. cit.*, p. 94.

<sup>4</sup> *Ibid.*, No. 39, 1945.

<sup>6</sup> *Ap. J.*, 99, 93, 1944.

and  $0.4 P$ , and peaked maxima near phase  $0.85 P$ . All these curves correspond to large values of  $e$  and to values of  $\omega$  in the first quadrant. There is no indication from the photometric observations that any of these stars have eccentric orbits, and the conclusion is that in all four stars the velocity-curves are deformed by the absorption in gaseous rings.

In deriving the values of  $T$  (periastron) for the elliptical orbits, I gave no consideration to the photometric observations. For the circular orbits I chose  $T$  (min. vel.) in such a way as to satisfy the epoch of mid-eclipse.

The following notes refer to the individual stars:

1. *SS Librae*.—The spectrum at maximum light is A5, and there is no appreciable change at principal eclipse. The velocity-curve shows a conspicuous rotational disturbance during the partial phases, and the direction of rotation is in the same sense as that of orbital motion. The spectrum shows only one component.

TABLE 3\*  
PHOTOMETRIC ORBITS

No.	Star	Ecl.	$L_b$	$r_b$	$k$	$i$	$\gamma$	Sol.	Sp <sub>b</sub>	Sp <sub>f</sub>	Ref.
2....	UU Oph	t	0.90	0.19	0.70	85°3	18.0	D	A0	(G2)	1
3....	W Sct	p	.79	.31	1.00	77.8	3.8	D	F?	(G5)	1
4....	RZ Dra	tp	.90	.34	0.77	72.0	15.0	U	F	(K4)	2
5....	U Sct	ap	.90	.42	1.47	83.0	4.2	D	G5	(K3)	2
6....	QY Aql	t	.94	.20	0.85	90.0	24.6	D	(gG2)	(gG9)	.....
7....	VW Cyg	t	.83	.12	0.54	90.0	17.0	D	A0	(F6)	2, 3
8....	AQ Peg	t	.92	.15	0.62	85.6	30.7	D	A2	(K1)	1
10....	WW And	t	0.64	0.08	0.75	89.3	3.1	D	A5	F3p	1, 4

\* The data for stars 2, 3, 4, 5, 7, 8, and 10 are taken from S. Gaposchkin, *Harvard Mono.*, No. 5, 1938. Those for star 6 are from B. S. Whitney, *Ap. J.*, **102**, 202, 1945. The references in the last column are: (1) S. Gaposchkin, *Veröff. Berlin-Babelsberg*, **9**, Part V, 1932; (2) H. Shapley, *Princeton Contr.*, **3**, 1915; (3) J. Fetlaar, *Bull. Astr. Inst. Netherlands*, No. 209, 1930; (4) A. B. Wyse, *Lick Obs. Bull.*, No. 464, 1934. The spectral types are taken from Gaposchkin and Whitney. Those in parentheses were computed by them from the values of  $\gamma$ .

TABLE 4  
LIST OF STAR LINES

Element	$\lambda$	No. of Star	Element	$\lambda$	No. of Star
Ca II.....	3933.67	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	Cr I+Ti II..	4290.03	6, 9
Fe I.....	4005.25	1, 5, 6, 7, 8, 9, 10	Ti II.....	4290.26	4, 8, 10
He I.....	4026.22	3, 11	Ti II+Fe II..	4302.50	4
Fe I.....	4045.82	1, 4, 5, 6, 7, 8, 9, 10	Ti II+Fe I..	4307.87	4, 7, 8
Fe I.....	4063.60	1, 4, 6, 7, 8, 9, 10	Sc II.....	4314.09	6, 7, 8, 9, 10
Ni II.....	4067.04	6, 9, 10	Sc II+Fe I..	4325.39	4, 7, 8
Fe I.....	4071.75	1, 4, 6, 7, 9, 10	H $\gamma$ .....	4340.47	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
Sr II.....	4077.71	1, 4, 6, 7, 8, 9, 10	Fe II.....	4351.77	5, 6, 7, 8, 9, 10
H $\delta$ .....	4101.74	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	Fe I.....	4383.55	1, 7, 8
Ti II+Fe II..	4172.70	1, 4, 6, 7, 8, 9, 10	Ti II.....	4395.04	1, 5, 6, 7, 8, 9, 10
Fe II.....	4178.85	1, 4, 6, 7, 8, 9, 10	Ti II.....	4399.77	4, 5, 6, 7, 8
Fe I.....	4199.13	9	Fe I.....	4404.77	7, 8
Sr II.....	4215.52	1, 4, 5, 6, 7, 8, 9, 10	Fe I.....	4415.13	7, 8
Ca I.....	4226.73	1, 4, 5, 6, 7, 8, 9, 10	Fe I.....	4435.01	7
Fe II.....	4233.16	1, 5, 6, 7, 8, 9, 10	Fe I+Ti II..	4443.09	1, 4, 5, 6, 7, 8, 9, 10
Fe I.....	4260.48	6, 7, 8	He I.....	4471.51	3, 11
Fe I.....	4271.48	7	Mg II.....	4481.23	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

## SPECTROGRAPHIC OBSERVATIONS

79

TABLE 5  
RADIAL VELOCITIES  
SS LIBRAE

PLATE G f/2	DATE 1945	U.T.	PHASE		VELOCITIES (Km/Sec) ALL LINES
			In Days	In Period	
5747.....	July 18	3:10	1.260	0.876	- 20.4
5757.....	19	3:02	0.816	.567	- 48.4
5771p*.....	20	2:46	0.367	.255	- 74.7
5785.....	21	2:59	1.376	.957	- 12.1
5797.....	22	2:54	0.935	.650	- 6.2
5811.....	23	3:00	0.501	.348	- 86.9
5822.....	24	2:59	0.062	.043	- 95.4
5836.....	25	2:59	1.062	.739	- 12.8
5899.....	30	2:44	0.300	.209	-100.1
5930p.....	Aug. 1	3:07	0.878	.611	- 11.1
5939.....	2	3:00	0.435	.303	- 88.6
5959.....	3	3:05	0.000	.000	- 92.7
5960.....	3	3:56	0.036	.025	- 91.7
5961.....	3	4:43	0.069	.048	- 85.1
5976.....	4	2:58	0.996	.693	+ 25.0
5994.....	5	2:34	0.541	.376	- 88.5
6010.....	6	2:59	0.120	.083	- 86.2
6024p.....	7	2:51	1.115	.775	- 29.1
6035p.....	8	3:41	0.711	.494	- 56.6
6049.....	10	3:07	1.250	.869	+ 3.1
6066.....	11	3:00	0.807	.561	- 41.4
6087.....	13	2:54	1.365	.949	- 21.9
6088.....	13	3:57	1.409	0.980	- 7.4

## UU OPHIUCHI

PLATE G f/2	DATE 1945	U.T.	PHASE		VELOCITIES (Km/Sec) ALL LINES
			In Days	In Period	
5750.....	July 18	5:19	0.247	0.056	+ 43.0
5761.....	19	5:06	1.237	.281	+ 10.0
5772.....	20	3:26	2.168	.493	+ 54.6
5786.....	21	3:43	3.180	.723	+ 79.0
5798.....	22	3:47	4.183	.951	+ 82.3
5812.....	23	3:33	0.777	.177	- 6.5
5823.....	24	3:42	1.783	.406	+ 25.8
5837.....	25	3:47	2.787	.634	+ 34.2
5916.....	31	3:10	4.364	.993	+166.8
5917.....	31	4:07	0.007	.002	+161.8
5918.....	31	5:20	0.057	.013	+139.7
5931.....	Aug. 1	3:43	0.990	.225	+ 34.0
5940.....	2	3:40	1.988	.452	+ 58.9
5977.....	4	3:37	3.986	.907	+ 54.2
5978.....	4	4:13	4.011	.912	+ 97.3
5995.....	5	3:38	0.589	.134	+ 23.5
6011.....	6	3:33	1.586	.361	+ 30.4
6025.....	7	3:25	2.580	.587	+ 40.6
6050.....	10	3:55	1.205	.274	+ 26.9
6067.....	11	4:00	2.209	.502	+ 32.2
6103.....	14	2:52	0.764	.174	+ 30.4
6104.....	14	3:36	0.795	.181	+ 18.7
6118.....	15	3:49	1.804	0.410	+ 13.3

\* The letters "p" and "vp" in the first column stand for "poor" and "very poor," respectively.

TABLE 5—Continued

## W SCUTI

PLATE	DATE	U.T.	PHASE		VELOCITIES (KM/SEC)		
			In Days	In Period	All Lines†	Ca II	H
CQ	1944						
3534.....	July 25	5:20	1.492	0.145	- 15.4	+ 8.7	+ 50.2
G f/2							
4666p.....	Aug. 11	4:45	8.197	.798	- 38.4	.....	- 38.4
4667.....	11	5:10	8.214	.800	- 64.9	+10.3	- 87.3
4675.....	12	5:32	9.230	.899	- 46.7	- 1.5	- 21.0
4685.....	13	4:25	10.183	.991	- 53.9	- 5.8	- 24.3
4686.....	13	5:27	10.226	.996	- 30.2	+ 1.8	- 2.3
4687.....	13	6:39	0.006	.001	- 39.0	.....	- 29.0
4693.....	14	5:22	0.953	.093	- 29.0	+ 9.2	+ 6.3
4694.....	14	6:11	0.987	.096	+ 12.2	+ 9.2	+ 53.8
CQ							
3624.....	20	4:22	6.911	.673	- 47.1	- 1.0	+ 53.7
3629.....	21	3:38	7.880	.767	- 162.2	.....	- 162.2
3648.....	26	5:16	2.678	.261	+ 23.0	.....	+118.0
3677.....	Sept. 1	3:16	8.595	.837	- 33.0	+ 8.0	- 65.5
3688.....	2	3:49	9.618	.936	+ 36.5	-13.5	+ 14.1
3704p.....	10	3:02	7.314	.712	-124.7	.....	-124.7
3715.....	11	2:51	8.307	.809	-125.0	-36.7	-125.0
3732p.....	14	2:44	1.032	.101	+ 27.5	.....	+ 27.5
3745.....	15	2:34	2.025	.197	+ 23.4	-21.6	+ 56.7
3757.....	16	3:10	3.050	.297	- 21.9	.....	+ 7.8
3778.....	21	3:05	8.046	.783	- 56.3	+14.4	- 54.6
G f/2							
4738.....	23	2:22	10.017	.975	+ 9.6	.....	+ 52.4
CQ							
3799p.....	28	4:13	4.828	.470	- 43.9	.....	- 35.2
3809.....	30	3:10	6.780	.660	- 57.1	+26.7	- 75.5
3821.....	Oct. 1	3:16	7.784	.758	- 88.8	+17.6	-115.0
G f/2	1945						
5763.....	July 19	6:15	1.069	.104	+ 50.3	+38.2	+ 49.9
5776.....	20	4:56	2.015	.196	+ 21.4	+15.8	+ 32.7
5790.....	21	5:55	3.056	.298	- 14.4	+11.6	+ 39.0
5802.....	22	5:18	4.030	.392	+ 30.5	.....	+ 40.9
5841.....	25	5:22	7.033	.685	- 67.3	- 5.5	- 87.0
5853.....	26	5:00	8.017	.781	- 80.9	.....	- 68.8
5866.....	27	5:11	9.025	.879	-105.4	+16.6	- 82.6
5932.....	Aug. 1	4:24	3.722	.362	- 15.1	+40.4	+ 56.1
5941.....	2	4:22	4.721	.460	- 20.0	+14.0	+ 2.0
5962.....	3	5:31	5.769	.562	- 44.2	- 1.8	- 35.9
5980.....	4	5:22	6.763	.658	- 70.9	+31.3	-102.0
5997.....	5	6:13	7.798	.759	- 52.7	- 2.6	- 83.9
6051.....	10	4:38	2.461	.240	+ 54.1	+29.2	+ 96.5
6052.....	10	5:18	2.489	.242	+ 63.2	+22.2	+ 89.8
6069.....	11	5:13	3.485	.339	+ 42.5	- 4.7	+118.3
6090.....	13	5:18	5.489	.534	- 15.2	+17.4	+ 0.3
6105.....	14	4:27	6.453	.628	- 44.0	+27.7	- 65.6
6119.....	15	4:40	7.462	.727	- 67.2	- 2.4	-100.9
6137.....	16	5:59	8.517	0.829	- 54.7	+ 8.6	+ 33.7

† All lines except Ca II.

TABLE 5—Continued

## RZ DRACONIS

PLATE G f/2	DATE 1945	U.T.	PHASE		VELOCITIES (KM/SEC) ALL LINES
			In Days	In Period	
5766.....	July 19	8:26	0.235	0.427	- 60.0
5779.....	20	7:31	.095	.172	-117.1
5791.....	21	6:50	.515	.935	+ 49.9
5792.....	21	7:40	.550	.998	- 80.9
5803.....	22	6:13	.380	.706	+113.7
5816.....	23	5:34	.260	.471	- 32.9
5826.....	24	4:48	.126	.229	-111.0
5827.....	24	5:21	.149	.270	-117.3
5843.....	25	7:06	.120	.218	-102.1
5854.....	26	5:47	.514	.933	+ 34.8
5868.....	27	6:31	.444	.806	+ 72.7
5879.....	28	4:42	.266	.483	- 18.7
5880.....	28	5:27	.297	.539	- 20.0
5881.....	28	7:40	.389	.706	+123.7
5882.....	28	8:21	.418	.759	+ 78.3
5883.....	28	9:04	.448	.813	+ 86.4
5888.....	29	6:01	.219	.398	- 67.6
5889.....	29	6:38	.244	.443	- 40.3
5890.....	29	7:19	.273	.496	- 19.4
5891.....	29	8:06	.306	.555	- 3.0
5908.....	30	7:23	.174	.316	-128.0
5920.....	31	7:24	.073	.133	- 97.1
5935.....	Aug. 1	9:51	.073	.133	- 84.1
5942.....	2	5:09	.327	.594	+ 44.0
5943.....	2	5:40	.348	.632	+ 81.0
5963.....	3	6:16	.271	.492	+ 22.4
5981.....	4	6:12	.166	.301	-111.7
5997.....	5	4:43	.004	.007	- 63.3
5998.....	5	5:20	.029	.053	- 67.4
6015.....	6	8:39	.065	.117	- 96.7
6042.....	9	8:00	.284	.516	- 26.7
6070.....	11	6:06	.001	.002	- 37.4
6080.....	12	6:02	.447	.811	+ 76.3
6081.....	12	7:00	.488	.886	+ 61.6
6082p.....	12	7:50	.522	.948	+ 23.1
6092.....	13	7:07	.391	.710	+ 95.5
6108.....	14	6:40	.271	.492	+ 0.7
6120.....	15	5:34	.123	.223	-129.4
6139.....	16	7:52	0.117	0.212	-115.5

## U SCUTI

PLATE G f/2	DATE 1945	U.T.	PHASE		VELOCITIES (KM/SEC) ALL LINES
			In Days	In Period	
5764.....	July 19	6:54	0.179	0.187	- 82.7
5777.....	20	5:37	.407	.426	-112.8
5842.....	25	6:17	.423	.443	- 17.7
5867.....	27	5:54	.497	.520	- 1.0
5933.....	Aug. 1	5:11	0.692	0.725	+ 73.2

TABLE 5—Continued

## QY AQUILAE

PLATE G f/2	DATE 1945	U.T.	PHASE		VELOCITIES (KM/SEC) ALL LINES
			In Days	In Period	
5751.....	July 18	6:45	6.931	0.959	+ 68.0
5767.....	19	9:21	0.811	.112	+ 8.8
5780.....	20	8:27	1.773	.245	- 12.2
5793.....	21	8:43	2.784	.385	+ 10.0
5804.....	22	7:13	3.722	.515	+ 40.1
5817.....	23	6:22	4.686	.648	+ 49.3
5828.....	24	5:54	5.667	.784	+ 61.7
5844.....	25	7:57	6.752	.934	+ 53.9
5855.....	26	7:02	0.484	.067	+ 18.5
5869.....	27	7:23	1.499	.207	- 8.2
5910.....	30	9:31	4.588	.635	+ 52.5
5921.....	31	7:27	5.501	.761	+ 77.9
5934.....	Aug. 1	6:11	6.449	.892	+ 56.5
5946p.....	2	7:23	0.270	.037	+ 32.8
5966.....	3	8:15	1.306	.181	+ 5.4
5984.....	4	8:13	2.304	.319	- 4.8
6000.....	5	7:17	3.265	.452	+ 22.2
6053.....	10	6:16	0.993	.137	- 2.5
6054.....	10	7:17	1.035	.143	- 2.8
6071.....	11	8:49	2.099	.290	+ 18.8
6094.....	13	8:38	4.092	.566	+ 50.9
6110.....	14	8:07	5.070	.701	+ 63.8
6123.....	15	7:51	6.059	0.838	+ 63.1

## VW CYGNI

PLATE G f/2	DATE 1945	U.T.	PHASE		VELOCITIES (KM/SEC)		
			In Days	In Period	All Lines	Ca II	H
5754.....	July 18	9:39	5.321	0.631	- 33.7	-26.1	-18.1
5768.....	19	10:07	6.341	.752	- 16.4	- 8.1	-25.1
5781p.....	20	9:15	7.304	.866	- 15.9	-15.9	.....
5805.....	22	8:01	0.823	.098	- 36.5	-57.6	-45.2
5819p.....	23	8:03	1.824	.216	- 38.6	-50.3	-43.7
5845.....	25	8:45	3.854	.457	- 45.8	-62.4	-55.0
5860.....	26	9:48	4.897	.581	- 41.6	-25.4	-46.8
5892.....	29	8:49	7.856	.932	- 5.3	+ 4.4	+14.0
5909.....	30	8:39	0.418	.050	- 37.6	-37.0	-26.7
5923.....	31	8:52	1.427	.169	- 47.0	-19.1	-29.0
5944.....	Aug. 2	6:10	3.315	.393	- 67.5	-49.2	-74.0
5945p.....	2	6:35	3.332	.395	-102.0	.....	-73.6
5964.....	3	6:55	4.346	.516	- 32.1	-57.2	-42.0
5965.....	3	7:26	4.368	.518	- 40.3	-49.6	-33.3
5982.....	4	6:48	5.341	.634	- 41.9	- 4.9	-56.5
5983.....	4	7:20	5.364	.636	- 20.6	-34.7	-44.4
6003.....	5	9:15	6.443	.764	- 12.5	+ 9.1	-45.2
6004.....	5	9:47	6.466	.767	+ 3.3	+ 6.1	- 1.2
6016.....	6	9:16	7.444	.883	- 11.7	+24.1	+14.8
6027.....	7	4:42	8.254	.979	- 3.6	.....	-22.4
6028.....	7	6:13	8.317	0.987	- 45.1	.....	-79.9

TABLE 5—Continued

VW CYGNI—Continued

PLATE G f/2	DATE 1945	U.T.	PHASE		VELOCITIES (Km/Sec)		
			In Days	In Period	All Lines	Ca II	H
6029.....	Aug. 7	8:15	8.402	0.997	- 17.5	.....	- 1.4
6030.....	7	10:16	0.056	.007	5.0	.....	43.6
6036.....	8	6:45	0.909	.108	55.4	-35.9	33.6
6037.....	8	7:37	0.945	.112	33.1	-43.5	25.6
6043.....	9	8:40	1.989	.236	58.1	-66.8	56.0
6057.....	10	9:10	3.010	.357	43.6	-48.0	56.3
6072.....	11	9:37	4.029	.478	39.5	-56.0	53.4
6079.....	12	4:55	4.833	.573	38.8	-56.0	66.6
6093.....	13	7:46	5.952	.706	15.3	-15.3	23.5
6109.....	14	7:17	6.931	.822	9.0	- 0.5	20.1
6121.....	15	6:23	7.894	.936	29.3	+ 3.2	19.7
6134.....	16	3:06	0.327	.039	64.6	-41.7	58.2
6135.....	16	3:49	0.357	0.042	- 64.2	-41.8	-56.9

## AQ PEGASI

PLATE G f/2	DATE 1945	U.T.	PHASE		VELOCITIES (Km/Sec)		
			In Days	In Period	All Lines	Ca II	H
5755.....	July 18	10:29	3.467	0.625	+ 1.8	-10.8	- 9.7
5782p.....	20	10:05	5.450	.982	+81.1	+78.4	+69.1
5806.....	22	8:24	1.832	.330	-44.0	-49.5	-44.7
5820vp.....	23	10:40	2.926	.527	-10.4	.....	.....
5846.....	25	9:25	4.874	.878	+35.7	+43.0	+49.9
5861.....	26	10:24	0.366	.066	-41.8	-32.1	-39.6
5871.....	27	8:39	1.293	.233	-16.7	-36.1	-50.8
5893.....	29	9:30	3.329	.600	+ 0.6	+11.6	-30.1
5911.....	30	10:15	4.360	.786	+33.0	+25.7	+24.2
5922.....	31	8:24	5.283	.952	+23.7	+36.9	+45.2
5947.....	Aug. 2	8:12	1.727	.311	-36.4	-46.2	-58.4
5948.....	2	8:42	1.747	.315	-38.6	-16.5	-24.8
5967.....	3	9:04	2.763	.498	-42.5	-31.6	-32.7
5968.....	3	9:34	2.784	.502	-41.8	-47.0	-54.8
5985.....	4	9:01	3.761	.678	+20.4	+19.9	+17.3
5986.....	4	9:20	3.774	.680	+13.5	-30.4	+35.8
6001.....	5	8:07	4.723	.851	+39.4	+12.8	+78.9
6002.....	5	8:33	4.741	.854	+21.3	+15.7	+12.8
6012.....	6	4:47	0.036	.006	- 3.3	.....	-15.6
6013.....	6	6:20	0.101	.018	-10.7	.....	-57.9
6014.....	6	7:36	0.154	.028	+ 1.7	.....	-36.6
6017p.....	6	9:58	0.252	.045	-90.3	.....	-30.7
6018.....	6	10:29	0.274	.049	-62.9	-48.2	-53.8
6044.....	9	9:38	3.238	.584	-16.4	+10.8	- 2.1
6055.....	10	8:05	4.174	.752	+20.4	+24.9	+36.2
6056.....	10	8:34	4.194	.756	+38.0	+21.1	+36.5
6073.....	11	10:14	5.263	.949	+15.4	+50.3	+19.6
6095.....	13	9:25	1.680	.303	-29.6	-43.6	-48.7
6111.....	14	8:57	2.661	.480	-30.0	-28.7	-22.9
6112.....	14	9:28	2.682	0.483	- 6.5	-14.2	-30.3

TABLE 5—Continued

AQ PEGASI—Continued

PLATE G //2	DATE 1945	U.T.	PHASE		VELOCITIES (Km/Sec)		
			In Days	In Period	All Lines	Cu II	H
6124.....	Aug. 15	8:39	3.648	0.657	-17.3	-18.4	-15.2
6140.....	16	8:33	4.644	.837	+15.9	+63.4	+38.5
6149.....	17	3:01	5.414	.976	+56.0	+70.6	+52.7
6150.....	17	4:08	5.460	.984	+78.1	+71.0	+86.7
6151.....	17	5:50	5.531	.997	+53.8	+74.6	+21.3
6152.....	17	7:54	0.069	.012	-3.5	.....	+1.7
6153.....	17	9:54	0.152	0.027	+20.7	.....	+25.9

## XX CEPHEI

PLATE G //2	DATE 1945	U.T.	PHASE		VELOCITIES (Km/Sec) ALL LINES
			In Days	In Period	
5756.....	July 18	11:08	2.323	0.994	-19.1
5809.....	22	11:13	1.652	.707	-3.7
5833.....	24	11:01	1.306	.559	-28.5
5834.....	24	11:12	1.314	.562	-35.5
5849.....	25	11:12	2.314	.990	-15.7
5858.....	26	9:09	0.891	.381	-40.5
5859.....	26	9:21	0.900	.385	-50.8
5875.....	27	11:01	1.969	.842	+4.9
5876.....	27	11:15	1.979	.847	-3.2
5896.....	29	11:09	1.638	.701	-2.1
5903.....	30	4:25	0.019	.008	-15.2
5904.....	30	4:56	0.041	.018	-43.4
5906.....	30	6:17	0.097	.042	-50.2
5907.....	30	6:36	0.110	.047	-33.7
5924.....	31	9:14	1.220	.522	-23.2
5953.....	Aug. 2	11:08	0.962	.412	-65.9
5954.....	2	11:19	0.970	.415	-50.5
5955.....	2	11:30	0.977	.418	-33.2
5971.....	3	11:21	1.971	.843	+3.1
5972.....	3	11:32	1.978	.846	-8.7
5989.....	4	11:20	0.633	.271	-63.6
5990.....	4	11:32	0.641	.274	-63.9
6007.....	5	11:30	1.640	.702	-4.8
6020.....	6	11:27	0.300	.128	-49.6
6031.....	7	11:30	1.302	.557	-15.5
6061.....	10	11:25	1.962	.839	-7.7
6062.....	10	11:37	1.970	.843	-5.7
6076.....	11	11:37	0.633	.271	-59.6
6098.....	13	11:34	0.293	.125	-46.2
6115.....	14	11:27	1.288	.551	-18.1
6116.....	14	11:39	1.296	.554	-21.5
6127.....	15	11:20	2.283	.977	-2.6
6142.....	16	9:44	0.880	.376	-56.7
6143.....	16	9:57	0.889	.380	-52.6
6155.....	17	11:35	1.957	0.837	-4.2

TABLE 5—Continued

## XY CEPHEI

PLATE G/f/2	DATE 1945	U.T.	PHASE		VELOCITIES (KM/SEC)	
			In Days	In Period	All Lines	Ca II
5808.....	July 22	10:33	0.632	0.228	- 58.2	-18.2
5848.....	25	10:47	0.867	.312	- 80.4	-44.6
5856.....	26	8:18	1.764	.636	+ 44.0	+19.6
5857.....	26	8:48	1.785	.643	+ 15.9	.....
5873.....	27	10:08	0.065	.023	- 50.8	.....
5874.....	27	10:38	0.086	.031	- 43.4	.....
5894.....	29	10:15	2.070	.746	+ 9.1	+ 0.6
5895.....	29	10:44	2.090	.753	+ 23.7	-21.5
5902.....	30	3:48	0.027	.010	- 46.1	.....
5905.....	30	5:40	0.105	.038	- 20.2	-25.4
5925.....	31	9:34	1.268	.457	- 56.7	-25.3
5950.....	Aug. 2	10:05	0.514	.185	-102.4	- 7.1
5951.....	2	10:30	0.532	.192	- 85.8	-25.4
5952.....	2	10:51	0.546	.197	- 74.0	-10.9
5970.....	3	10:58	1.551	.559	+ 25.7	+12.0
5988.....	4	10:56	2.550	.919	+ 31.4	-17.6
6006.....	5	11:10	0.784	.283	- 53.0	-21.4
6019.....	6	11:05	1.781	.642	+ 19.2	+ 0.6
6045.....	9	10:36	1.987	.716	+ 35.0	-21.5
6059.....	10	10:36	0.212	.076	- 29.5	-10.8
6060.....	10	11:03	0.230	.083	- 10.3	-21.5
6074.....	11	10:50	1.221	.440	- 29.9	- 7.1
6075.....	11	11:16	1.239	.447	- 25.0	-21.6
6097.....	13	11:05	0.458	.165	- 59.5	-25.5
6114.....	14	11:04	1.457	.525	- 0.1	+ 4.2
6126.....	15	10:42	2.442	.880	+ 26.3	-14.9
6141.....	16	9:17	0.608	.219	- 59.6	.....
6154.....	17	11:13	1.688	0.608	+ 38.9	.....

## WW ANDROMEDAE

PLATE G/f/2	DATE 1945	U.T.	PHASE		ALL LINES	Ca II	Fe I	VELOCITIES (KM/SEC)		H	Ti II	Ca I	Ni II	Mg II	Sc II
			In Days	In Period				Fe II	Sr II						
4901.....	Jan. 25	2:15	6.024	0.259	- 3.2	-20.7	+ 5.8	+42.6	.....	-18.6	- 31.2	- 32.2	.....	-16.3	.....
4916.....	26	1:47	7.004	.301	+ 9.1	-80.9	+47.9	+ 2.9	+ 24.3	-19.7	.....	+ 35.2	.....	- 4.3	.....
4947.....	28	2:18	9.026	.388	+ 4.5	-24.3	+14.3	+19.5	+ 30.2	-27.7	- 63.8	+ 21.2	.....	+18.7	.....
4957.....	29	1:56	10.011	.430	-26.7	-39.3	+ 4.1	+ 2.0	-43.1	-44.3	- 52.1	+ 41.0	.....	-56.8	.....
5020.....	Feb. 2	2:55	14.052	.603	-35.9	- 9.1	-57.7	-37.6	+ 3.7	- 6.6	-48.0	-40.5	.....	-55.6	-17.9
5769.....	July 19	11:07	18.398	.790	-43.9	- 8.2	-29.2	-43.5	.....	-31.6	-152.1	-41.8	.....	-19.6	-68.4
5783.....	20	10:58	19.392	.833	-39.1	+10.7	-96.4	-20.5	+ 28.7	-41.5	.....	-114.3	.....	-77.3	-37.2
5794.....	21	9:46	20.342	.874	-44.6	+18.4	-58.0	-44.9	-17.6	-31.0	- 66.8	- 85.1	.....	-59.8	-42.7
5807.....	22	9:25	21.327	.916	-28.4	+29.0	-39.8	- 7.0	-48.1	+ 0.9	-47.3	-123.6	.....	-59.9	+ 8.7
5818.....	23	7:25	22.244	.955	-33.1	.....	-40.3	+29.5	.....	-54.8	-44.8	-104.4	.....	-43.1	-21.9
5829.....	24	6:53	23.222	.997	-11.0	-19.1	-17.8	-10.8	- 5.6	- 5.8	-28.6	- 8.7	+52.2	-72.0	+33.6
5830.....	24	7:58	23.267	.999	-32.6	-15.3	-27.9	-20.2	-49.7	-18.6	- 67.6	- 37.8	- 6.1	-43.2	.....
5831.....	24	9:10	0.032	.001	-29.8	-45.8	-20.1	-26.8	-46.3	-29.0	-46.7	- 18.6	- 6.1	-54.6	.....
5832.....	24	10:19	0.080	.003	- 9.7	- 8.5	- 2.6	+ 4.1	-23.0	-20.9	-20.9	+ 27.9	.....	+26.3	- 1.9
5847.....	25	10:09	1.073	.046	- 8.1	-19.3	+ 8.3	- 7.8	- 1.1	- 8.2	-32.2	+ 38.0	.....	- 2.6	-37.5
5862.....	26	11:01	2.109	.091	+ 8.6	-19.4	.....	+11.4	-51.2	- 9.0	.....	+ 34.3	.....	+38.1	+43.7
5872.....	27	9:25	3.042	.131	- 4.3	- 8.7	+ 8.6	+ 4.9	.....	-14.0	-48.9	- 4.0	.....	-26.1	+28.4
5912.....	30	11:01	6.109	.262	+14.8	-23.7	+11.9	+49.0	-34.8	-16.8	-17.4	+ 67.8	.....	+55.0	+43.2
5926.....	31	10:08	7.072	.304	+25.0	.....	+68.7	.....	-18.7	-10.8	.....	+ 52.9	.....	.....	.....
5949.....	Aug. 2	9:25	9.042	.388	- 0.7	-65.2	+30.0	+12.7	+15.9	-29.5	+ 4.9	-28.9	.....	-38.3	.....
5969.....	3	10:17	10.078	0.433	-15.2	-47.0	+ 1.8	- 2.8	- 5.9	-17.8	- 58.8	- 9.9	.....	-49.8	-32.4

TABLE 5—Continued  
WW ANDROMEDAE—Continued

PLATE G/f/2	DATE 1945	U.T.	PHASE		ALL LINES	Ca II	Fe I	VELOCITIES (KM/SEC)		H	Ti II	Ca I	Ni II	Mg II	Sc II
			In Days	In Period				Fe II	Sr II						
5987.....	Aug. 4	10:14	11.076	0.476	- 8.6	- 2.2	-22.4	-12.7	+ 9.4	-12.8	.....	- 39.2	.....	+ 7.6	+12.0
6005.....	5	10:30	12.088	.519	18.3	.....	-14.8	-17.0	- 27.3	-32.6	- 51.0	39.2	.....	+25.0	+53.0
6058.....	10	9:56	17.064	.733	41.6	+ 8.2	-45.4	-58.8	+ 1.2	- 0.8	79.1	87.7	.....	-10.8	-65.4
6096.....	13	10:16	20.078	.862	33.0	+19.0	-64.1	-38.9	.....	+15.0	35.5	64.2	.....	-56.6	- 9.7
6113.....	14	10:18	21.079	.905	28.7	+29.5	-35.7	-13.9	-104.8	+14.6	68.7	74.3	.....	-23.0	+ 5.4
6125.....	15	9:39	22.052	.947	24.5	+33.1	+20.2	-17.1	- 82.7	-14.1	60.6	55.4	.....	-34.6	-20.6
6136.....	16	4:51	22.852	.981	17.2	-18.7	-39.1	+10.5	- 29.3	-19.8	42.9	36.2	.....	-34.6	+71.9
6144.....	16	10:53	23.103	0.992	- 5.0	-22.8	+ 7.8	+ 1.7	-42.5	+ 1.2	-14.7	- 7.4	+27.8	+28.7	-46.6

TABLE 6  
ELEMENTS OF SPECTROGRAPHIC ORBITS

No.	Star	P (Photo- m.) (Days)	$\gamma$ (Km/Sec)	$K_1$ (Km/Sec)	$e$	T (Phase of Min. Vel.)	T (Phase of Peri- astr.)	$\omega$ (De- grees)	$\frac{m_3^3}{(m_1+m_2)^2} \sin^3 i \odot$	$a_1 \sin i$ (Km)
1..	SS Lib	1.44	-45	45	0	0.25 P	.....	.....	0.014	$0.89 \times 10^6$
2..	UU Oph	4.40	+50	30	0	.25 P	.....	.....	.012	$1.82 \times 10^6$
3..	W Sct	10.27	-16	76	0	.75 P	.....	.....	.458	$10.5 \times 10^6$
4..	RZ Dra	0.55	-16	104	0	.25 P	.....	.....	.081	$0.79 \times 10^6$
5..	U Sct	0.95	-14	86	0	.25 P	.....	.....	.063	$1.13 \times 10^6$
6..	QY Aql	7.23	+34	36	0.06	.....	0.25 P	180	.035	$3.57 \times 10^6$
7..	VW Cyg	8.43	-30	34	.18	.....	.90 P	15	.033	$3.88 \times 10^6$
8..	AQ Peg	5.55	- 8	35	.24	.....	.85 P	15	.023	$2.59 \times 10^6$
9..	XX Cep	2.34	-29	28	0.14	.....	0.00 P	76	.005	$0.88 \times 10^6$
10..	WW And	23.28	-17	27	0	.75 P	.....	.....	.048	$8.64 \times 10^6$
11..	XY Cep	2.77	-14	54	0	0.25 P	.....	.....	0.045	$2.06 \times 10^6$

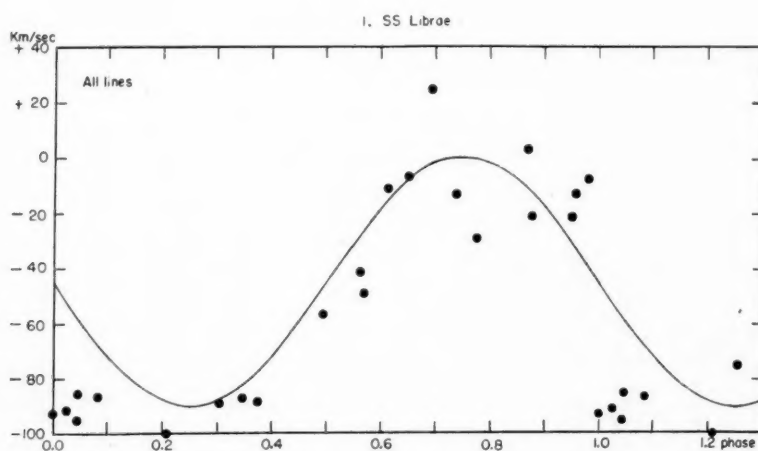


FIG. 1

2. *UU Ophiuchi*.—This star has a faint visual companion. The range at principal minimum appeared to be less than 2.4 mag. (see Table 1), and the star was never difficult to observe. The spectral type is A0 and is not changed at principal eclipse. At maximum light the lines are very broad. Hence, it is not surprising that there should be a large rotational disturbance. The velocity-curve shows large positive residuals at phases 0.993, 0.002, and 0.013, but at phase 0.056 *P* the velocity is normal. I suspected from my estimates that the minimum came later than had been predicted. Hence, it is probable that I have observed only the first half of the eclipse. At phase 0.002 *P* the lines *Ca II K* and *Hγ* appear unsymmetrical, being shaded on the violet sides. Only one component can be observed.

3. *W Scuti*.—This star is an unusually difficult object. Its visual magnitude at maximum is about 9.5, but photographically it is more nearly of magnitude 10.3. The spectrum is of type B3n. The lines are exceedingly diffuse and faint. There is no change in spectrum at principal minimum. The broad interstellar absorption band at  $\lambda 4430$  is very strong, and the star is undoubtedly greatly reddened by interstellar dust. The interstellar line *Ca II K* gives a mean velocity of +8.8 km/sec. The *He I* lines look double on many spectrograms, but I was unable to measure the components separately. The *H* lines appear single on most spectrograms, though on a few there was strong suspicion of the presence of a relatively weak secondary component. The measures of the *H* lines should be almost unaffected by blending. Those of the *He I* lines are probably strongly influenced. Hence the orbit was derived from the *H* lines. It is of interest that in this system the more luminous component stands in front during the principal minimum. We conclude that the secondary is a star of perhaps type B0, while the primary is a star of type B3. In the light of this information it will be necessary to revise the photometric orbit.

4. *RZ Draconis*.—This star, which has a faint visual companion, is interesting because of its short period and unequal minima. The spectrum is of class A5 with moderately diffuse lines, and the type does not change at the two eclipses. But the lines become much stronger in the vicinity of principal mid-eclipse. This may be due to the sharpening of the lines produced by the rotational effect. The velocity-curve suggests a small rotational disturbance between phases 0.9 *P* and 1.1 *P*. There does not seem to be any appreciable change in spectrum from phase 0.2 *P*, through phase 0.5 *P* and to phase 0.8 *P*. Hence there is no evidence of a difference in temperature at the corresponding hemispheres of the brighter component.

5. *U Scuti*.—The spectrum is of type F0. Since only five spectrograms were obtained, the velocity-curve is quite uncertain, and even the range is considered as a rough approximation. Only one component has been observed.

6. *QY Aquilae*.—The spectral type at maximum light is F0, and the lines are fairly narrow. No observations were made during the total eclipse. Only one component has been observed.

7. *VW Cygni*.—The spectral type in full light is A3 with strong lines of *H*, *Fe II*, and *Ca II* and weaker lines of *Fe I*, *Ca I*, *Mg II*, *Sr II*, *Ti II*, etc., all in absorption. This spectrum closely resembles that of RW Per.<sup>7</sup> The lines are somewhat broadened by rotation or turbulence. The *H* lines have strong Stark-effect wings. The principal eclipse lasts about 24 hours, or 0.12 *P*. The total phase lasts 8 hours or 0.04 *P*. During the partial eclipse the metallic lines become more conspicuous and sharper than outside of eclipse. This is noticeable at phase 0.936 *P* and again at 0.042 *P*. There are no emission lines at these phases. The spectral type remains about A3 until phase 0.979 *P*, when the absorption lines of *Fe I* and *Ca I* are greatly strengthened and the G band makes its appearance. At this phase I suspect faint red emission borders at the *H* lines. At phase 0.987 *P* the absorption lines suggest a type of about G5 with very strong and deep lines of *Fe I*,

<sup>7</sup> Struve, *op. cit.*, Pl. III, facing p. 76.

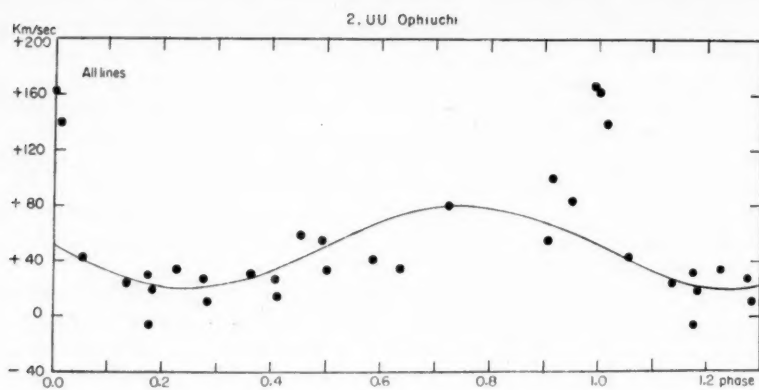


FIG. 2

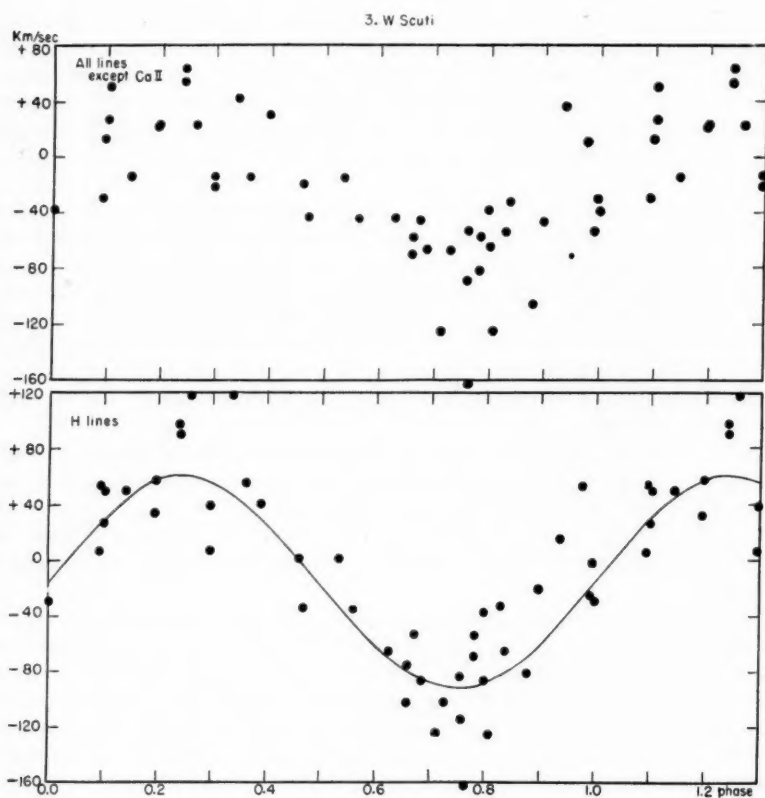


FIG. 3

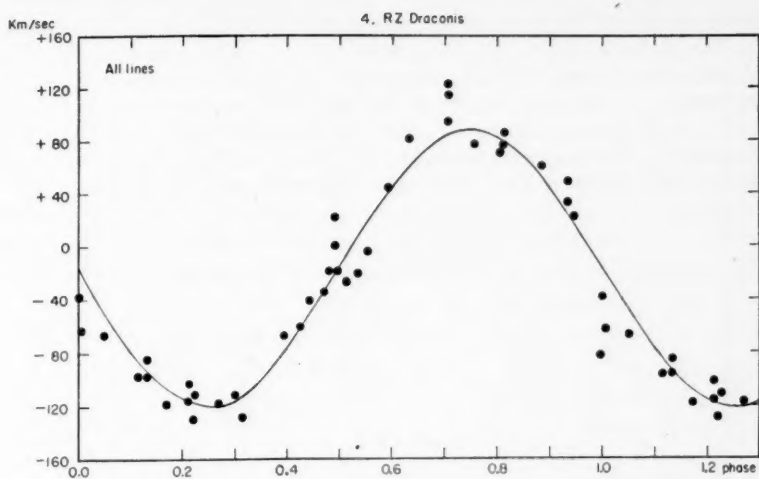


FIG. 4

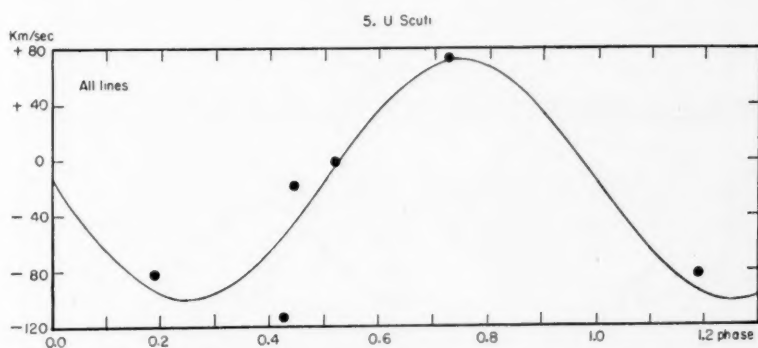


FIG. 5

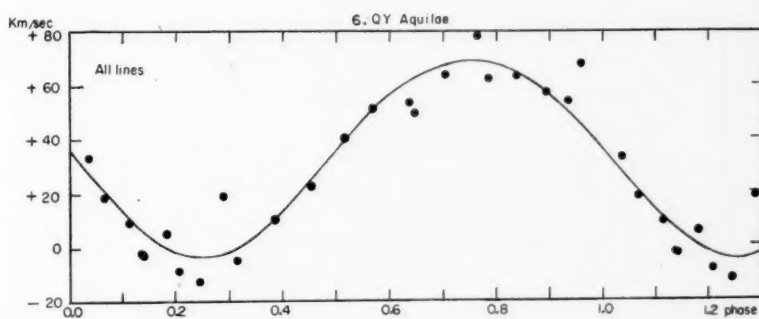


FIG. 6

$\text{Ca I}$ , and probably  $\text{Sr II}$ . The spectrum at this stage suggests greater than main-sequence luminosity. The  $H$  lines,  $\text{Mg II } 4481$ , and probably  $\text{Ca II K}$  and several  $\text{Fe II}$  lines have strong and rather broad red emission components. The violet components are either absent or very weak. These lines, especially  $H$  and  $\text{Mg II}$ , have strong absorption cores which cannot be attributed to the G5 star. At phase 0.997  $P$ , near mid-eclipse, the emission lines are quite weak and probably consist of two symmetrical components. It is probable that the emitting ring is almost completely eclipsed at this stage. The G5

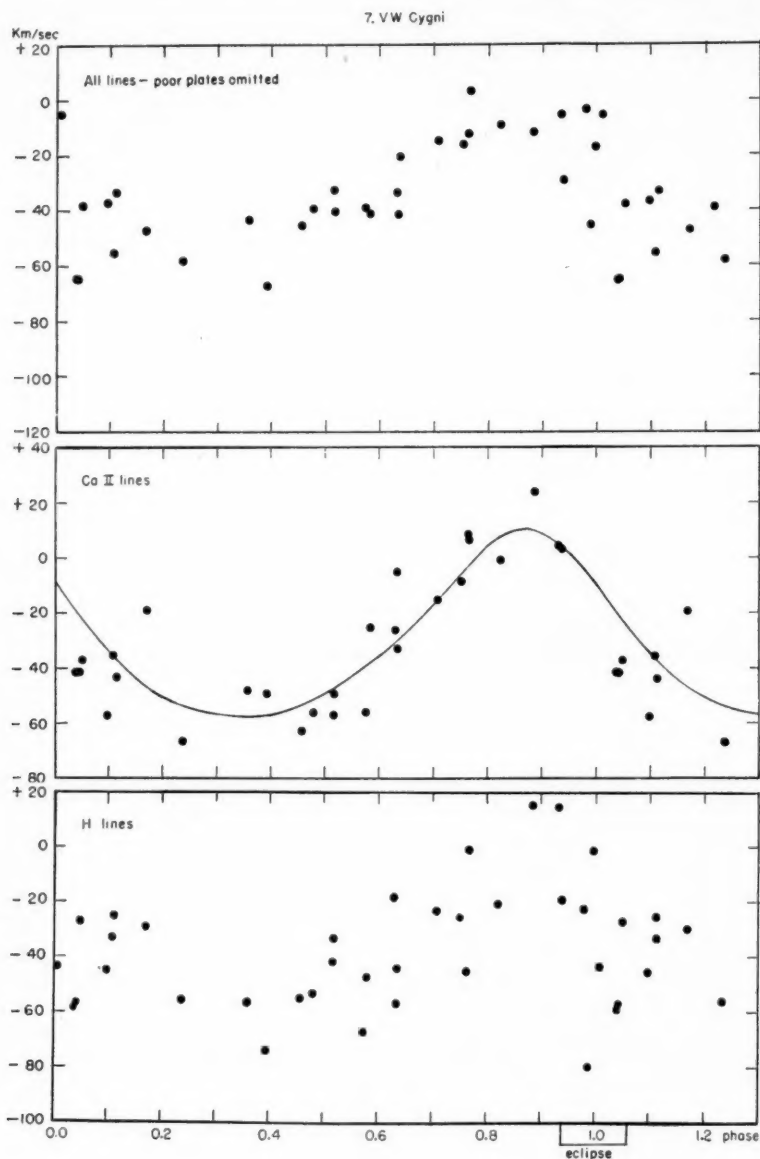


FIG. 7

absorption spectrum remains unchanged. At phase 0.007  $P$  the violet emission components are present, though they are not so strong as the violet components were at phase 0.997  $P$ , and the red components are absent. The absorption spectrum is still of type G5, but I have the impression that the lines are not so deep as before mid-eclipse, and the spectrum no longer suggests great luminosity.

The velocity-curve is unsymmetrical, and the broad, shallow minimum at about phase 0.36  $P$  is unmistakable. The scatter is larger than I should have expected, and it is not improbable that there are real changes from day to day. The maximum at phase 0.86  $P$  is not appreciably affected by the eclipse, but the very steep descent of the curve between phases 0.9  $P$  and 0.1  $P$  may be caused, in part, by a rotational disturbance.

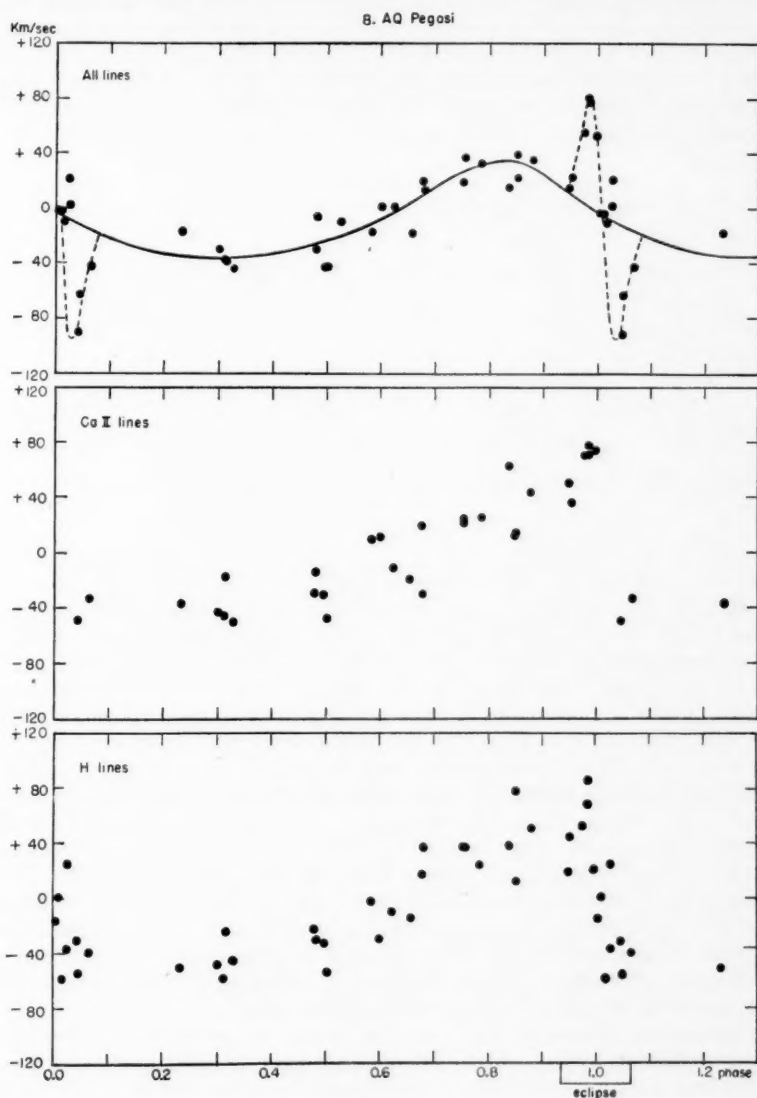


FIG. 8

Although this disturbance is not identifiable as such, it is clearly present in RW Per and AQ Peg. The elements were derived from the  $Ca$  II lines alone. These lines also gave an excellent velocity-curve in the case of RW Per. In both stars the  $H$  lines gave a much greater amount of scatter than the  $Ca$  II lines or the means of all lines.

8. *AQ Pegasi*.—In full light the spectrum is of type A2, with very strong absorption lines of  $H$  and  $Ca$  II and with weak lines of  $Fe$  II,  $Mg$  II,  $Fe$  I,  $Ti$  II,  $Ca$  I, etc. The spectrum resembles those of VW Cyg and RW Per. In all three stars the lines of  $Fe$  II are relatively strong—a feature which is usually associated with a higher luminosity than is found in the case of A-type stars having weak lines of  $Fe$  II. The lines of  $Ti$  II are somewhat stronger in AQ Peg than in VW Cyg. The lines of AQ Peg are broadened by rotation. The  $H$  lines have strong Stark-effect wings. They, and the  $Ca$  II lines, may have absorption cores produced by turbulence broadening. I have suspected that these cores vary in intensity, as they do in the case of RW Per, but neither in the case of AQ Peg nor in that of VW Cyg could such a variation be established beyond doubt. The principal eclipse lasts about 18 hours, or  $0.135 P$ . The total phase lasts 5 hours, or  $0.038 P$ . The middle of eclipse occurs about 2 hours, or  $0.015 P$ , later than was predicted from the ele-

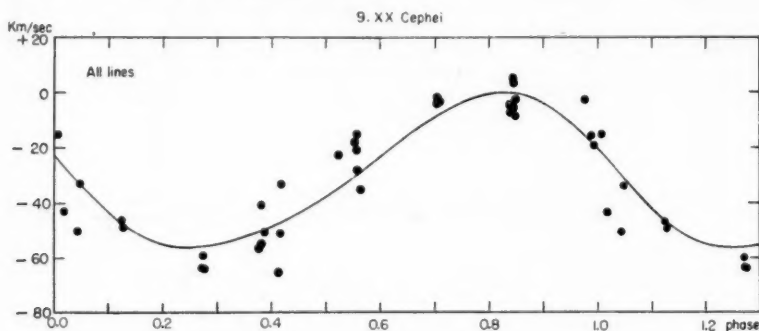


FIG. 9

ments in Table 2. Therefore, the eclipse actually took place between phases  $0.948 P$  and  $0.082 P$ , while totality lasted from  $0.996 P$  to  $0.034 P$ . The metallic absorption lines become sharp and conspicuous during the partial eclipse. This is noticeable at phase  $0.982 P$  and pronounced at  $0.984 P$ . It is probably present at phases  $0.045 P$ ,  $0.049 P$ , and  $0.066 P$ , but my spectrograms at the first two of these phases are not strong enough to be certain. There are no emission lines at these four phases. At phase  $0.997 P$  the absorption spectrum of type A2 is blended with one of much later type, and at phase  $0.012 P$  it is of type G5 with strong and very deep lines of  $Fe$  I and  $Ca$  I, closely resembling the G5 spectrum of VW Cyg. At phase  $0.997 P$ , strong red emission components are present at  $H$ ,  $Mg$  II,  $Ca$  II, and probably  $Fe$  II. There is no definite evidence of violet emission components. At phase  $0.012 P$  the emission lines are very weak, and they consist of two unequal components, the red being the stronger. At phase  $0.027 P$  the violet emission components are strong, and there are no red components. At all these phases the lines of  $H$ ,  $Mg$  II,  $Ca$  II, and probably  $Fe$  II have central absorption cores. From the emission lines we should infer that mid-eclipse took place a little after phase  $0.012 P$ —in good agreement with the estimates of magnitude, which gave  $0.015 P$ . The absorption spectrum at phase  $0.027 P$  was still of type G5.

The velocity-curve is unsymmetrical, the broad and shallow minimum at phase  $0.35 P$  being unaffected by the eclipse. There is, however, in this star a striking rotational disturbance, best shown by the means of all lines. I believe that while the effect is not very clearly shown in the measures of the  $Ca$  II and  $H$  lines, it is, nevertheless, present

in all three curves, giving rise to the very steep descent of the curve between phases 0.98  $P$  and 0.02  $P$ . This result is of importance: it shows that the lines of the A2 object really belong to the hotter component and that the gases producing them are eclipsed by the G5 star. This conclusion agrees with that reached in my study of RW Per. The spectrographic elements were derived from the means of all lines.

9. *XX Cephei*.—The spectral type is about A8, with fairly strong lines of *Fe I*, *Ca I*, *Sr II*, etc., and normal lines of *Fe II*, *Ti II*, and *Mg II*. The spectrum does not change at principal minimum. The exposures were usually about 10 minutes in duration. Since

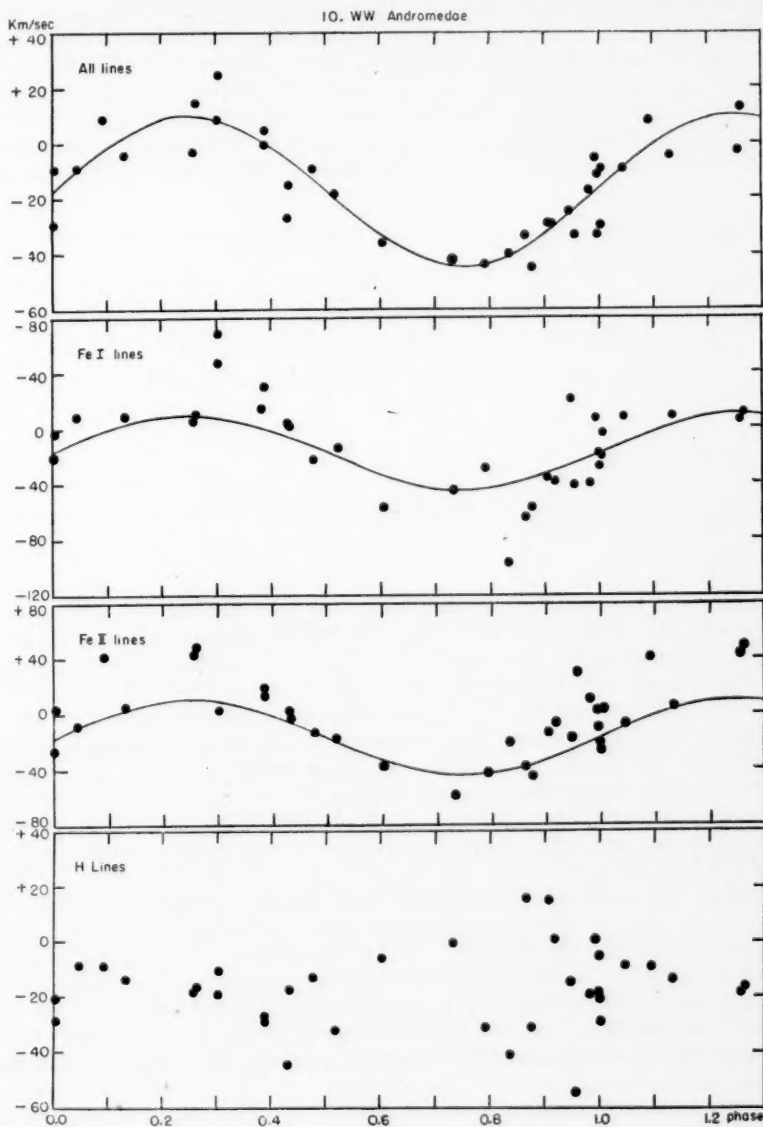


FIG. 10

strong, widened spectra were obtained with these short exposures, I suspect that the photographic magnitude may be somewhat brighter than 8.7, as given by Schneller. Only one component is seen in the spectrum.

10. *WW Andromedae*.—Of all the stars on this program, *WW And* seemed to be a priori the best candidate for *H* emission lines: A. B. Wyse<sup>8</sup> had found that in its secondary spectrum "the Balmer lines are progressively weaker toward the less refrangible region, indicating that *H $\alpha$*  is probably bright." My spectrograms do not show *H $\alpha$* , but they otherwise confirm Wyse's observations, without, however, definitely showing bright lines at *H $\beta$* , *H $\gamma$* , or *H $\delta$* . During the eclipse the type is F3, with unusually sharp and narrow absorption lines of *H*. Outside of eclipse the type is A5. But the most remarkable feature of the spectrum is the line *Ca II K*. This line is stronger and broader in the A5 spectrum than in the F3 spectrum, though at times (phases 0.733 *P*–0.862 *P*) it looks like a narrow line superposed over a broader one. The eclipse lasts 40 hours, or 0.07 *P*. Totality lasts 7 hours, or 0.012 *P*. The spectrum is normal at phases 0.947 *P* and 0.955 *P*. Even at phase 0.981 *P* the A5 spectrum predominates. At this phase there is a faint suspicion of a red emission component at *H $\beta$* . At phase 0.992 *P* there is no emission at any

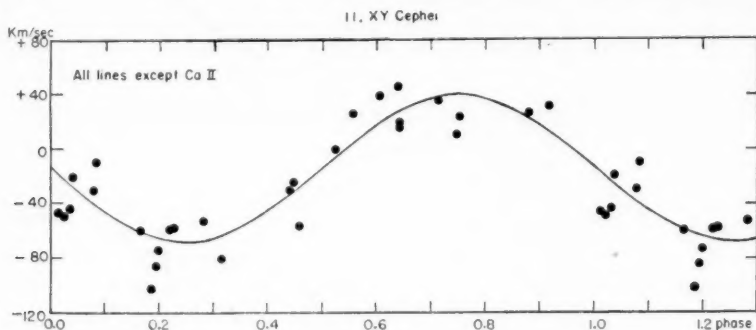


FIG. 11

of the *H* lines, and the spectrum is now mostly of type F3, but *H $\delta$* , and especially *H $\epsilon$*  and *H $\zeta$*  are still too broad and too strong to be wholly attributed to the F3 star; *H $\gamma$*  is narrow, and *H $\zeta$*  is very weak. At phase 0.997 *P* the spectrum is entirely F3, and *Ca II K*, *H $\beta$* , *H $\gamma$* , *H $\delta$* , and *H $\zeta$*  are strikingly narrow and devoid of wings. It is interesting that *H $\epsilon$*  and *Ca II H* can be resolved. There is no clear indication of emission at any of these lines, but only a vague suspicion of red emission borders at *Ca II K* and *H $\beta$* . These may not be real, but the corelike appearance of the absorption lines is not in doubt. At phase 0.999 *P* the absorption lines are unchanged, but there is no longer even a suspicion of red emission borders. The same is true at phases 0.001 *P* and 0.003 *P*, but at the latter phase *H $\beta$*  may be double. At phase 0.046 *P* the spectrum is blended, and at phase 0.091 *P* it is again normal.

The velocity-curves are complicated and show that the two spectra are badly blended. The means of all lines are strongly influenced by *Fe I*, *Fe II*, *Sr II*, and *Ca I*, all of which belong predominantly to the F3 star. This star is in front during the principal eclipse. The elements in Table 6 were determined from the means of all lines. The lines of *Ca II* show a slight trend that is opposite to that of the rest of the lines, while *H* and *Mg II* show almost no variation. I believe that the lines of *H* and *Ca II* are badly blended but that in them the A5 component predominates, while in *Mg II* the F3 component predominates.

The question of the emission lines in this star remains unanswered; the evidence,

<sup>8</sup> *Lick Obs. Bull.*, 17, 41, 1934.

though qualitatively in agreement with that obtained in other stars, is quite insufficient, and it will be necessary to use red-sensitive plates for  $H\alpha$ . But the corelike appearance of  $Ca II$  and  $H$  in the F3 star is very strange. It is consistent with the earlier observations in SX Cas, RW Per, etc., all of which seemed to show, during the total eclipse, absorption cores produced by shell-like gaseous masses *in front* of the eclipsing star.

11. *XY Cephei*.—The spectral type is B8 in full light and perhaps A0 at mid-eclipse. The lines of  $He I$  and  $Mg II$  are exceedingly diffuse and faint. Only one component is shown in the spectrum. Since an exposure time of 25 minutes gave strong, widened spectra, I believe that the photographic magnitude at maximum is somewhat brighter than 10.0, as given by Schneller. The  $Ca II$  K line is interstellar, with a mean velocity of  $-13.8$  km/sec.

## SYSTEMATIC ERRORS

In order to determine the systematic errors of my measurements, I have measured 28 spectrograms of  $\alpha$  Boo obtained under the same conditions as were the spectrograms of the program stars. The results are given in Table 7. The radial velocity of  $\alpha$  Boo as

TABLE 7  
RADIAL VELOCITIES OF  $\alpha$  BOOTIS

Plate G f/2	Date 1945	U.T.	Velocities (Km/Sec)	Plate G f/2	Date 1945	U.T.	Velocities (Km/Sec)
5770.....	July 20	2:25	-10.9	6021.....	Aug. 7	2:11	- 1.6
5784.....	21	2:23	-11.9	6032.....	8	2:09	+ 0.6
5796.....	22	2:21	- 0.6	6046.....	10	2:13	-12.6
5810.....	23	2:34	-15.4	6063.....	11	2:06	-10.4
5821.....	24	2:23	- 2.2	6083.....	13	1:56	- 6.8
5835.....	25	2:20	- 8.6	6084.....	13	1:58	-15.4
5850.....	26	1:17	-11.6	6099.....	14	2:01	-16.6
5897.....	30	2:17	- 1.7	6100.....	14	2:03	-14.0
5898.....	30	2:20	+ 2.5	6128.....	16	2:05	- 3.5
5913.....	31	2:15	- 7.2	6129.....	16	2:07	- 3.5
5927.....	Aug. 1	2:14	-10.7	6130.....	16	2:09	- 8.4
5936.....	2	2:14	-10.9	6145.....	17	1:56	-19.6
5956.....	3	2:14	-10.4	6146.....	17	1:59	-12.9
5973.....	4	2:13	- 7.3				
5991.....	5	2:12	- 7.0	Mean.....			- 8.8 $\pm$ 0.7

determined by Adams with the Mount Wilson coude spectrograph<sup>9</sup> is  $-5.6$  km/sec. We therefore have the following difference:

$$\text{McDonald} - \text{Mount Wilson} = -3.2 \pm 0.7 \text{ km/sec.}$$

The corresponding correction has not been applied to the measures in Table 5. The probable error of one observation of  $\alpha$  Boo with the G f/2 arrangement is  $\pm 3.7$  km/sec, in good agreement with my previous experience.

The wave lengths of the star lines as used in  $\alpha$  Boo and as adjusted to fit the average radial velocity of  $-8.8$  km/sec are given in Table 8.

## DISCUSSION

It is now possible, for the first time, to discuss the properties of binary stars possessing ringlike gaseous structures. Table 9 contains all the relevant data for eight stars which fall into this class. I have included TT Hyd, which is discussed elsewhere in this issue

<sup>9</sup> *Ap. J.*, 93, 14, 1941.

by Sahade and Cesco, and also RZ Oph, which has been observed during the present season by Hiltner, Sahade, Edmondson, and the writer. I am indebted to Dr. Hiltner for the information that the bright lines of this star also undergo eclipses. The spectral type of RZ Oph is listed in the literature as cG0 + gK5p, but I believe that the hotter component is considerably earlier, perhaps cF5.

The following results have been obtained:

1. The emitting rings occur over a vast range in period, from 2.77 to 261.9 days.

TABLE 8  
WAVE LENGTHS OF LINES IN  $\alpha$  BOOTIS

Element	Preliminary $\lambda$	Correction	Corrected $\lambda$	Probable Error
Fe I.....	4005.25	+0.37 A	4005.62	$\pm 0.01$ A
Fe I.....	4045.82	— .02	4045.80	.02
Fe I.....	4063.60	+ .12	4063.72	.02
Ni II.....	4067.04	+ .17	4067.21	.02
Fe I.....	4071.75	+ .02	4071.77	.02
Sr II.....	4077.71	— .56	4077.15	.02
H $\delta$ .....	4101.74	— .46	4101.28	.02
Ti II+Fe II.....	4172.70	+ .03	4172.73	.02
Sr II.....	4215.52	+ .08	4215.60	.02
Ca I.....	4226.73	— .05	4226.68	.01
Ti II.....	4290.26	— .22	4290.04	.02
Sc II+Fe I.....	4325.39	+ .18	4325.57	.02
Fe I.....	4383.55	+0.35	4383.90	$\pm 0.02$

TABLE 9  
ECLIPSING BINARIES WITH GASEOUS RINGS

Star	$P$	$A_1$	$A_2$	$D$	$d$	Spectrum	$K_1$ (Km/Sec)	$V_{em}$ (Km/Sec)	Character of Eclipse of Ring
RW Tau....	2.77	3 <sup>m</sup> 4	0 <sup>m</sup> 0	9 <sup>h</sup>	1 <sup>h</sup>	B9+K0	.....	350	Total; em. only at eclipse
AQ Peg....	5.55	2.7	.1	18 <sup>h</sup>	5 <sup>h</sup>	A2+G5	35	290	Total; em. only at eclipse
TT Hya....	6.95	2.5	.0	15 <sup>h</sup>	6 <sup>h</sup>	A3+G6	45	270	Annular; em. only at eclipse
VW Cyg....	8.43	2.6	.0	24 <sup>h</sup>	8 <sup>h</sup>	A3+G5	34	290	Total; em. only at eclipse
RW Per....	13.20	2.2	.0	24 <sup>h</sup>	10 <sup>h</sup>	A5+G0	18	220	Annular; em. only at eclipse
RX Cas....	32.3	0.7	.6	5 <sup>d</sup>	p	A3+G3	36	150	Annular; em. at all phases
SX Cas....	36.6	1.0	.3	5 <sup>d</sup>	1 <sup>d</sup> 2	A6+G6	50	150	Annular; em. at all phases
RZ Oph....	261.9	0.8	0.0	16 <sup>d</sup>	8 <sup>d</sup>	cF5+K5	.....	120	Annular; em. at all phases

2. The spectral types of those components which are eclipsed simultaneously with the rings range from B9 to F5, and there seems to be a close correlation between type and period.

3. The character of the double  $H$  emission lines in these stars is similar to that of the bright  $H$  lines in Be stars. Undoubtedly, in both kinds of stars they are produced by rapidly rotating gaseous rings.<sup>10</sup> But in the Be stars the overwhelming majority occurs in the earlier subdivisions of class B and in the O stars. In the binaries the phenomenon is associated mostly with spectral class A. This difference is extremely significant. There are a great many close binary systems in which at least one component is of early B type. With the exception of a few abnormal systems in which the bright lines do not resemble those discussed in this paper ( $\beta$  Lyr, HD 163181), these stars do not show emission lines.

<sup>10</sup> In the case of the Be stars this hypothesis was put forward by me in *Ap. J.*, **73**, 100, 1931.

It is probable that in suitable B-type systems the  $H$  in the rings is ionized and we fail to observe the bright lines. But the degree of ionization of the ring must be proportional to the ratio of the dilution factor to the density,  $W/\rho$ , or to  $1/r^2\rho$ , where  $r$  is the radius of the ring. Since the ionization of the rings in binaries is approximately the same as in Be stars, while the temperatures of the ionizing stars are very different, we should expect to find that  $r^2\rho$  is smaller in the case of the binaries. But it is difficult to see how either  $\rho$  or  $r$  can be very small, without causing the total emission from the ring to become so small that it would be unobservable in full light.

4. There is an interesting correlation between the velocity of rotation of the ring and the period of the binary. Disregarding the last star, for which the value of  $V_{em}$  is preliminary, we find that we have a close relation of the form

$$V_{em}^3 \sim P^{-1}.$$

This is in form identical with Kepler's third law. But we must remember that the masses of the stars are not all the same and that the periods are those of the binary components and not those of the rings. Nevertheless, the relation shows that, as the periods increase, the dimensions of the rings increase roughly in accordance with Kepler's third law.

TABLE 10  
SOME ECLIPSING BINARIES WITHOUT RINGS

Star	$P$	$A_1$	$A_2$	$D$	$d$	Spectrum
U Cep.....	2.49	2.4	0.1	10 <sup>h</sup>	2 <sup>h</sup>	A0+G8
SX Hya.....	2.90	4.0	.0	8	1	A3+K
UU Oph.....	4.40	2.4	.0	12	2	A0
U Sge.....	3.38	2.9	0.1	13	2	B9+G2

5. The eclipses of the rings tend to be total for the short-period systems and annular for the long-period systems. This means that in the smaller systems the rings are relatively smaller in size than the eclipsing stars.

6. The total emission from the rings is also a function of  $P$ . In the long-period binaries the bright lines are visible even in full light, while in the short-period binaries the bright lines can be seen only during very deep total eclipses.

7. The spectra of the hotter components are interesting because they belong to A stars with relatively strong  $Fe\ II$  lines and not to the so-called metallic-line A stars. Since the spark lines are enhanced in the more luminous stars, we conclude that high luminosity favors the occurrence of bright lines.

8. But the stars of shorter period are not supergiants. There is a gradual increase in luminosity with period in Table 7.

9. We can now specify the necessary requirements for observing the rings: the period must be longer than about 2.5 days; the eclipses must be total and must be very deep for the shorter periods but need not be especially deep for the longer periods; and the spectral types must be between B9 and F5.

10. Since relatively few eclipsing stars meet all conditions, we infer that ring formation is a fairly common phenomenon.

11. But there are striking cases in which no bright lines have been observed, despite the fact that they have been looked for. A few such cases are listed in Table 10. These examples, as well as our new observations of WW And, suffice to show that the rings are not all identical.

12. In two stars, SX Cas and RX Cas, the emission lines vary in character throughout the light-curve; the central absorption core disappears at certain phases. This shows that

we are not concerned with undisturbed circular motion around the A-type components. As one might have expected, the rings present a complicated dynamical system, and it is not at present possible to describe the motions accurately from the observations.

13. The question of whether we are actually concerned with rings or with spherical shells cannot be definitely answered from the observations alone. A thin spherical shell rotating with constant angular velocity around a very small star gives a contour

$$I(\Delta\lambda) = \text{constant}.$$

If the star is not small, the contour shows an indentation; and if the star is of approximately the same size as the shell, the contour becomes

$$I(\Delta\lambda) = \frac{\text{const.}}{\sqrt{1 - \Delta\lambda^2}}.$$

This is also the expression for the contour of a rotating ring with very small star. But a ring with relatively large star should give two isolated components, with no emission between. It is probably not advisable to rely too much upon the observational data, because the latter are influenced by such unexplained phenomena as the strong core absorptions during total eclipse. But it is perhaps significant that in the short-period systems the bright components appear far apart from one another, without emission between them. I believe that observationally we can better rely upon the absorption phenomena in Be stars, such as  $\phi$  Per,  $\zeta$  Tau, Pleione, etc. They strongly favor the ring hypothesis.

14. An important difference between ordinary Be stars and binaries with rings consists in the fact that the former all have rapid axial rotations, while the binaries are not, as a rule, characterized by excessive rotations. In fact, the absorption lines of SX Cas and RZ Oph are exceptionally sharp.

15. The preceding paragraph suggests that the origin of the rings is not the same in the two types of objects: in the Be stars we suspect that the rings are produced by rotational breakup. In the binaries the rings must have been produced at the time when the systems were formed. The ring is probably the debris that was scattered about when the original catastrophe took place.

16. Since relatively few binaries have sufficiently large inclinations to produce total eclipses, there must be other systems, not known as variables, which possess gaseous rings. It is possible that some of Merrill's Ae and Fe stars belong to this group. But it is perhaps significant that ordinary spectroscopic binaries of spectral class A rarely, if ever, show bright  $H$  lines similar to those observed in SX Cas and RX Cas. It is possible that the rings are characteristic features of those systems which we designate as "typical Algols" and which consist of a small, massive, early-type primary and a large, less massive, late-type secondary. The great bulk of the ordinary spectroscopic systems are altogether different in character.

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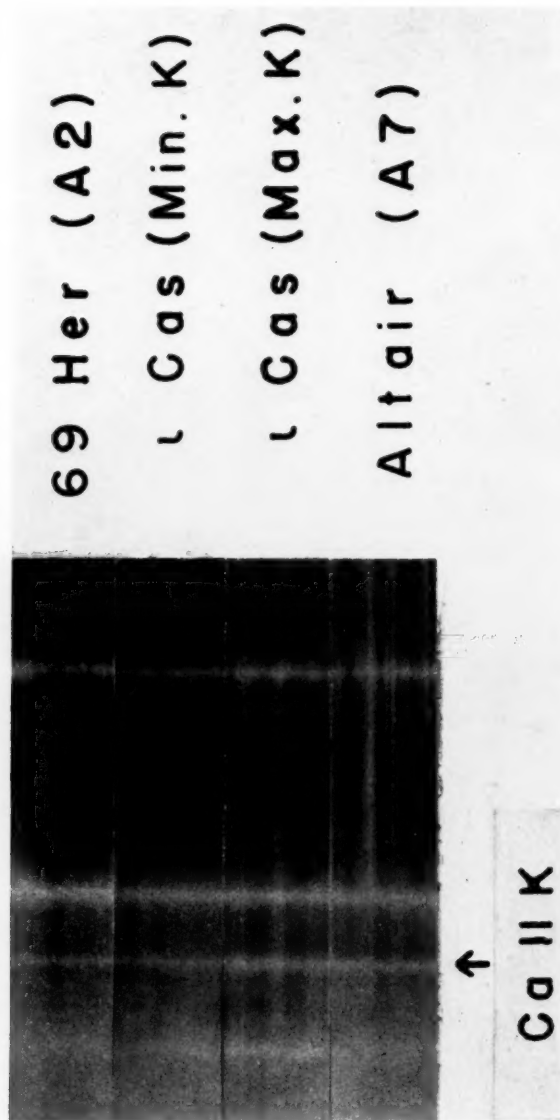
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PLATE VI



SPECTRA OF  $\iota$  CASSIOPEIAE IN COMPARISON WITH SPECTRA OF TWO NORMAL A STARS

In spectra of  $\iota$  Cas, note change in ratio of Ca II K to Ca II H + H $\epsilon$

## THE PERIOD OF THE SPECTRUM VARIABLE $\iota$ CASSIOPEIAE

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### ABSTRACT

In the spectrum of the chromium star  $\iota$  Cassiopeiae the lines of  $\text{Ca II K}$  and  $\lambda$  4078 are found to vary in intensity with a period of 1.740 days. The variation in the K line is equivalent to a change in spectral type from A2 to A7.

In the course of a spectroscopic survey of the brighter peculiar A stars, large changes were noted from night to night in the K line of  $\iota$  Cassiopeiae. The variability of this line was first pointed out by W. W. Morgan,<sup>1</sup> who found that the intensity varied between 11 and 15 on a scale in which 16 is the intensity of the K line in the F5 star Procyon. Morgan had earlier<sup>2</sup> detected rapid variations in the intensities of  $\lambda$  4549 and  $\lambda$  4558.

In the *Henry Draper Catalogue* the type of  $\iota$  Cas (HD 15089) is given as A5p, with the remarks that the line  $\lambda$  4078 is strong and that a second classification, from another plate, was A2p. Morgan<sup>3</sup> has characterized the star as belonging to the group of chromium stars, in which the lines of both  $\text{Cr II}$  and  $\text{Sr II}$  are strong. The prototype of this group is the well-known spectrum variable 73 Draconis.

The star  $\iota$  Cas has two companions, and all three components have common proper motion of  $0''.014$  toward  $349^\circ.5$ . The nearer companion (B), at a distance of  $2''$ , is 2.26 mag. fainter than the primary star; the more distant companion,  $7''$  away, is at least a full magnitude fainter than B. The orbits are unknown. Russell<sup>4</sup> gives the Mount Wilson types of A and B, respectively, as A3sp and F5; his dynamical parallax is  $0''.014$ , of "good" quality. In the *New General Catalogue* Aitken gives the Mount Wilson classifications A, cA4; B, F6; and C, G4; his dynamical parallax is  $0''.020$  from measures of AB and  $0''.022$  from measures of AC. The magnitude of the primary is 4.6, so that it is probably slightly above the main sequence.

From measures of five Yerkes spectrograms Frost, Barrett, and Struve<sup>5</sup> derived radial velocities which varied between  $-21$  and  $+10$  km/sec, with the mean at  $-2.8$  km/sec.

The recent observations were made at the 40-inch telescope with the 6-inch camera attached to the small, one-prism spectrograph. The dispersion was about 125 Å/mm at  $H\gamma$ ; exposure times were about 3 minutes. Eastman 103a-O plates were used throughout, and special effort was made to obtain plates of equal density and contrast.

The magnitude of the variation is illustrated in the reproductions of Plate VI. At minimum strength the K line is about as strong as in the A2 dwarf 69 Herculis, while at maximum it is stronger than in the A7 dwarf Altair. On each of 39 plates two independent estimates of the intensity of the K lines were made on a scale of 1-7. At 4 the K line looks about equal to  $H\delta$ ; at 7 it is much stronger than  $H\delta$  and approaches the intensity of  $H\epsilon + \text{Ca II H}$ . The mean residual of a single intensity estimate is 0.23 unit. The estimates are listed in Table 1, and the plate means are plotted in Figure 1.

<sup>1</sup> *Pub. Yerkes Obs.*, **7**, 133, 1935.

<sup>2</sup> *Ap. J.*, **76**, 275, 1932.

<sup>3</sup> *Ibid.*, **77**, 330, 1933.

<sup>4</sup> *The Masses of the Stars*, Chicago: University of Chicago Press, 1940.

<sup>5</sup> *Pub. Yerkes Obs.*, **7**, 1, 1929.

Estimates of the strength of the peculiar feature at  $\lambda$  4078 were made independently on the same 39 plates (plus one more). There is no other feature which could be compared with  $\lambda$  4078, so the estimates are less free from scatter than those of the K line. The estimates are listed in Table 1, and the plate means are plotted in Figure 2.

The elements adopted for the maximum intensity of the K line are

Maximum of K line at JD 2431700.1 + 1.740 E.

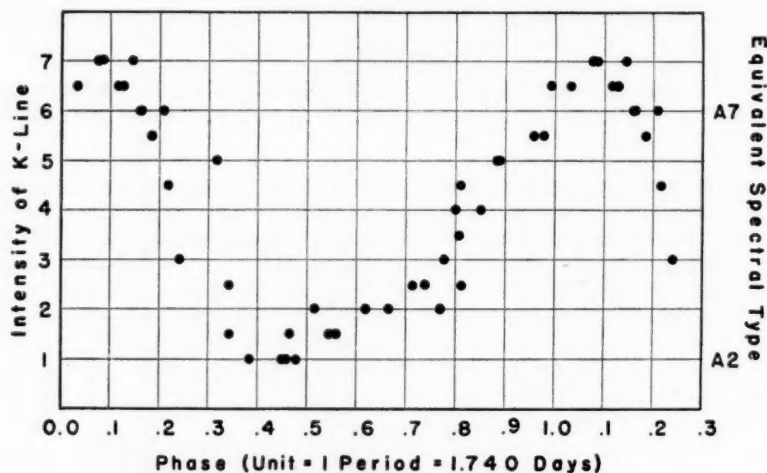


FIG. 1.—Intensity variation of the K line in the spectrum of  $\epsilon$  Cas

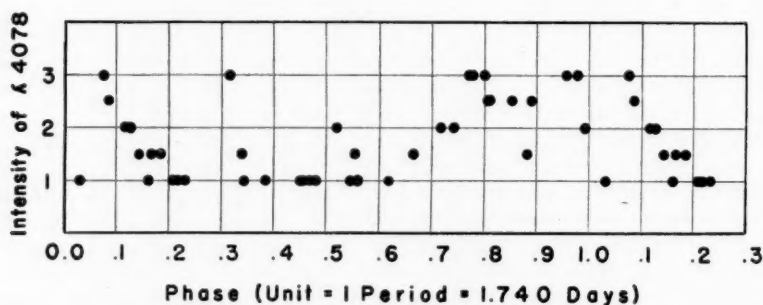


FIG. 2.—Intensity variation of the line  $\lambda$  4078 in the spectrum of  $\epsilon$  Cas

The observations extend over 37 cycles, so that the period should be correct to well within 1 per cent. The K line falls to minimum strength more rapidly than it rises to maximum, and the minimum is broader than the maximum. The same asymmetry appears in the curves, by Morgan<sup>6</sup> and Miss Durham,<sup>7</sup> that give the variation of the line  $Eu$  II 4205 in 73 Dra.

The line  $\lambda$  4078, which is due principally to  $Cr$  II and  $Sr$  II, varies in phase with  $Ca$  II K, and the same asymmetry is probably present in its variation. The figures suggest that  $\lambda$  4078 may reach its maximum shortly before the K line. Comparison of plates taken

<sup>6</sup> *Ap. J.*, 77, 77, 1933.

<sup>7</sup> *Ap. J.*, 98, 504, 1943.

near maximum strength of K with others taken near minimum suggest that certain other features in the spectrum vary in phase with K, notably the  $Cr$  II lines which appear on the red wing of  $H\delta$  and the line  $Sr$  II 4215. In order to provide material for a more complete descriptive study of the spectral variations, the star is currently being observed under somewhat higher dispersion. These observations should also lead to improved values for the period and for the epochs of maximum and minimum strength of K.

TABLE 1  
ESTIMATES OF LINE INTENSITIES IN THE SPECTRUM OF  $\epsilon$  CASSIOPEIAE

PLATE	JD 2431700+	PHASE*	INTENSITY ESTIMATES			
			K Line		$\lambda$ 4078	
A30.....	48.78	0.029	6	7	1	1
A2.....	1.88	.075	7	7	3	3
A11.....	17.55	.086	7	7	3	2
A8.....	15.86	.115	6	7	2	2
A21.....	31.54	.126	6	7	3	1
A18.....	24.61	.144	7	7	2	1
A28.....	45.52	.161	6	6	1	1
A9.....	15.95	.167	6	6	2	1
A33.....	52.52	.184	6	5	2	1
A37.....	57.78	.207	6	6	1	1
A25.....	36.92	.218	4	5	1	1
A13.....	19.56	.242	3	3	1	1
A27.....	38.83	.316	5	5	3	3
X969.....	5.81	.340	2	3	2	1
A35.....	54.54	.345	1	2	1	1
A5.....	14.59	.385	1	1	1	1
A6.....	14.70	.449	1	1	1	1
A14.....	19.93	.455	1	1	1	1
A24.....	35.61	.465	1	2	1	1
A48.....	66.95	.477	1	1	1	1
A10.....	16.56	.518	2	2	2	2
A41.....	61.85	.546	2	1	1	1
A17†.....	23.58	.552			1	2
A19.....	28.81	.558	2	1	1	1
A43.....	63.72	.620	2	2	1	1
A45.....	65.54	.666	2	2	2	1
A22.....	32.56	.714	3	2	2	2
A36.....	56.97	.741	3	2	2	2
X964.....	4.82	.770	2	2	3	3
A26.....	37.89	.776	3	3	3	3
A47.....	65.77	.800	4	4	3	3
A31.....	51.86	.805	3	4	3	2
A38.....	58.83	.810	4	5	3	2
A15.....	20.55	.811	2	3	2	3
A23.....	32.80	.850	4	4	3	2
A12.....	18.93	.880	5	5	1	2
A40.....	58.97	.891	5	5	3	2
A7.....	15.58	.955	5	6	3	3
A16.....	22.58	.976	6	5	3	3
A29.....	46.97	0.995	7	6	2	2

\* Phase = 0 at JD 2431700 + 1.740 E.

† K line underexposed; no estimate made.

I wish to thank Dr. W. W. Morgan for suggesting this investigation and to acknowledge his kind assistance and that of my wife in its execution.

## REVIEWS

*The Comet of 1577: Its Place in the History of Astronomy.* By C. DORIS HELLMAN. New York: Columbia University Press, 1944. Pp. 488. \$6.00.

This book is, beyond question, a valuable contribution to the history of astronomy even if only by the amount of material collected in it and systematized. Yet to the reviewer the book is an excellent illustration of a fallacy still current in some educational circles that method is of more importance than the factual content of the study.

It would seem fairly obvious that information about comets should be looked for in the astronomical literature, but this is decidedly not the attitude adopted by the author. The first two chapters of the book are devoted to the history of comets; but in spite of 517 references used in them, the reviewer has a definite impression of superficiality. The author is plainly unaware of the tremendous amount of work done by astronomers on the history of Halley's comet, especially on its apparition of 1456, when the comet was supposedly excommunicated by the pope, Calixtus III. At least, the fundamental work by J. Stein, S.J., *Calixte III et la comète de Halley*, is not mentioned; and the author is forced to speculate who Platina (responsible for the myth) was and what he wrote about. Researches by G. Celoria on the comets observed by Toscanelli (from 1433 to 1472) are briefly mentioned, but the reference is to an abstract in the *Astronomische Nachrichten*, and not to Celoria's detailed investigation in the *Pubblicazioni Brera-Milano*, which is the model for such researches. The monumental work of Holetschek is not mentioned at all. It seems that for the author the history of comets ends with Pingré's book published in 1783.

The result is an utterly inadequate history of comets, without the slightest attempt to separate the wheat from the chaff. Pre-telescopic observations of eclipses, novae, and comets are often very valuable from the purely scientific point of view; and by no means all that was written by ancient authors about comets is sheer nonsense. For instance, the various systems of classification of comets, beginning with Ptolemy, are based on good observation; and we may say that most bright comets go through the stages Pithei, Discei, Acontiae, and Xiphiae with diminishing heliocentric distance.

The remaining two-thirds of the book are devoted to the comet of 1577; but here, again, the impression is that the author is treating the comet as if it were a myth, whose origin and development she wants to study. Yet, it was a remarkable natural phenomenon, as a reference to Holetschek would have shown very easily. Undoubtedly it was one of the brightest comets on record—at least one hundred times as bright as Halley's comet.

The author accepts Peter Apian's dictum that the tails of comets are directed opposite to the sun a bit too literally. In fact, she praises Tycho Brahe and other astronomers for not wasting their time on the tail. However, Tycho Brahe's observations were used by Brandes and Bredichin (neither of whom is mentioned) to show that the main tail of the comet was of the second type (in Bredichin's classification) and that a side tail was of the third type. There was even an anomalous tail directed toward the sun, Peter Apian notwithstanding. The author mentions observations of this tail by Cornelius Gemma and Bazelius but passes over them as plainly erroneous.

Indeed, Tycho Brahe is the main theme of the book, but why Tycho Brahe's observations were so much better than those of other astronomers is not at all evident. From the perspective of nearly four hundred years we can see that Tycho Brahe's failure to observe a parallax of this comet was a turning-point in the history of comets, but this was not so clear to his contemporaries and successors. If Tycho Brahe solved the problem of comets by showing that the comet of 1577 was supralunar, why, then, did Galileo revert to the old theory that comets were exhalations in the atmosphere of the earth? Nearly one hundred years after the comet of 1577, Jean-Dominique Cassini, the first director of the Paris Observatory, still maintained that comets revolve around the earth, basing his theory on his own observations.

The rest of the book is devoted to the observations and opinions of Tycho Brahe's contemporaries: Maestlin, Roeslin, the Landgrave of Hesse, Cornelius Gemma, and a host of other astronomers, astrologers, poets, preachers, etc., with a large number of references and bibliograph-

ical and biographical data. The Appendix consists of 112 pages listing all known tracts and treatises on the comet of 1577, not including those in the Chinese and Arabic. The list consists of 111 items, many of them multiple, which bear witness to the interest aroused by the comet.

Comets and eclipses can be treated from the point of view of the impression produced by them, and it is quite possible (and perhaps desirable) to have a book on the sociological significance of comets. Yet, in a book which purports to be a contribution to the history of astronomy (as is clear from its subtitle), such an attitude is deplorable. In fact, with the present development and specialization of science, it is doubtful that a book of this sort should even be attempted without the close co-operation of a professional astronomer.

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*Perkins Observatory*

*Sonne und Erde.* By M. WALDMEIER. Zürich, Switzerland: Büchergilde Gutenberg, 1945. Pp. 239.

This semipopular book was written while the author was "far away from science, writing desk and library, mostly in military service and high among the mountains." Successive chapters are entitled as follows: (i) "Nature and Science"; (ii) "The Source of the Sun's Energy"; (iii) "The Radiation of the Sun"; (iv) "The Chemistry of the Sun"; (v) "The Appearance of the Sun"; (vi) "The Riddle and the Mystery of the Corona"; (vii) "The Sun's Cycle"; (viii) "The Sun, the Ionosphere and the Radio"; (ix) "The Sun, Terrestrial Magnetism and the Aurora Borealis"; (x) "The Sun, the Weather and the Climate." The printing is excellent, and there are many fine illustrations from the author's own work at Zürich and Arosa.

*Atomic Spectra and Atomic Structure.* By GERHARD HERZBERG. 2d rev. ed. New York: Dover Publications, 1944. Pp. xiv+257. Figs. 80. \$2.75.

The first edition of this book, which was reviewed on page 220 of Volume 87 of this *Journal*, has been out of print for some time. In this new edition Professor Herzberg has adopted Birge's new set of fundamental constants and has brought up to date a number of tables, especially the table of ionization potentials.

*Astronomy, Vol. I: The Solar System.* By H. N. RUSSELL, R. S. DUGAN, and J. Q. STEWART. Rev. ed. Boston: Ginn & Co., 1945. Pp. xi+470+xxi+pl. 1+183 figs. \$3.00.

The first edition of this famous textbook appeared in 1926.<sup>1</sup> The new edition contains many important changes, such as those "arising from improved methods of navigation; from later data concerning the internal constitution and age of the earth; from Jones's accurate determination of the solar parallax (which slightly alters a great number of numerical data); from new observations of the planets and their satellites—especially the discovery of Pluto—and from new physical studies of them; from similar work upon comets and meteors; and from recent radical changes in cosmogonic theory."

<sup>1</sup> Reviewed in this *Journal*, 67, 274, 1928.

## NOTICE

*Annales d'astrophysique*.—The French National Center of Scientific Research (C.N.R.S.) has recently announced that, beginning with 1946, the *Annales d'astrophysique* will be reorganized as an international medium for the publication of articles on astrophysics. Dr. F. Joliot Curie, director of the C.N.R.S., has nominated a French committee to direct the new *Annales*: chairman, Mr. Danjon, director of the Paris Observatory; vice-chairman, Mr. Chalonge, astronomer of the Paris Observatory and at the Institute of Astrophysics; secretary, Mr. Mineur, astronomer of the Paris Observatory and director of the Institute of Astrophysics; members, Messrs. Coulomb, Dufay, Fehrenbach, Lyot, Couder, Lallemand, and Barbier. Dr. Barbier has been appointed editor-in-chief. An international board of foreign correspondents has also been formed, including Messrs. Abetti, Beals, Gaviola, Lindblad, Mayall, Menzel, Merrill, Oort, Plaskett, Redman, Russell, B. Strömgren, Struve, and Swings. Professor Fessenkov was also chosen but has not yet replied. Under the new plan the *Annales* will accept articles from all countries and will be prepared to print them in one of three languages: French, English, or German.

During the war the *Annales* published two issues per year. Those for 1945 are expected to appear in the early part of 1946. The first issue of 1946, which will start the international series, is expected to appear in the middle of the year. The editor-in-chief expects that before long the number of issues will be increased to three per year; and, if circumstances should warrant, there may ultimately be as many as six or eight issues per year. The address of the editor is Dr. D. Barbier, Institut d'astrophysique, 98 Bis, Boulevard Arago, Paris 14E, France.

## ERRATA

The following corrections should be made in Volume 101:

On page 256, footnote 5, omit "21 and."

On page 258, line 31 should read "On Callisto the spots are hard to define because the disc has a low albedo."

On page 259 the first paragraph refers to Mars.

On page 259, footnote 8 should read "C.R., 219, 21, 1944."

On page 259, footnote 9, the value 0.750 for Titan refers to 9, not 9.54 astronomical units; the diameter is therefore 4890 km.

On page 260 the term "faculae," as used, refers to bright patches on K3 spectroheliograms, not on direct photographs.

On page 260, Section 4, read "Mr. H. Grenat" instead of "Mr. H. Greenat."

On page 260, Section 4, read "Lyot's coronagraph" instead of "Lyot's chronograph."